Computer Science Engineering

Q. No. 1 – 25 Carry One Mark Each

1. Let X be a Gaussian random variable mean 0 and variance \( \sigma^2 \). Let \( Y = \max(X, 0) \) where \( \max(a, b) \) is the maximum of \( a \) and \( b \). The median of \( Y \) is ____________.

**Key:** (0)

**Exp:**
‘\( X \)’ is Gaussian random variable
\[ X \sim N(0, \sigma^2) \text{ for } -\infty < x < \infty \]
Given \( y = \max(x, 0) \)
\[ y = \begin{cases} 0 & \text{if } -\infty < x \leq 0 \\ x & \text{if } 0 < x < \infty \end{cases} \]
is a random variable
Since median is positional average
Therefore, median of \( Y \) is '0'.

2. Consider the Karnaugh map given below, where \( x \) represents “don’t care” and blank represents 0.

Assume for all inputs (\( a, b, c, d \)) the respective complements \( \bar{a}, \bar{b}, \bar{c}, \bar{d} \) are also available. The above logic is implemented 2-input NOR gates only. The minimum number of gates required is ____________.

**Key:** (1)

**Exp:**

![Karnaugh Map Diagram]
F(a, b, c, d) = \overline{ac} \Rightarrow a + c = a + c \\
(\overline{x + y} = \overline{x}, \overline{y})

Only 1 NOR gate required

3. The statement \((\neg p) \Rightarrow (\neg q)\) is logically equivalent to which of the statements below?
   I. \(p \Rightarrow q\)
   II. \(q \Rightarrow p\)
   III. \((\neg q) \lor p\)
   IV. \((\neg p) \lor q\)

   (A) I only  
   (B) I and IV only 
   (C) II only 
   (D) II and III only

Key: (D)

Exp: By rule of contrapositive,
\[
\neg p \rightarrow \neg q \Leftrightarrow q \rightarrow p
\]
\(q \rightarrow p \Leftrightarrow \neg q \lor p\)

4. Consider the following table:

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Design Paradigms</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Kruskal</td>
<td>i. Divide and Conquer</td>
</tr>
<tr>
<td>Q. Quicksort</td>
<td>ii. Greedy</td>
</tr>
<tr>
<td>R. Floyd-Warshall</td>
<td>iii. Dynamic Programming</td>
</tr>
</tbody>
</table>

Match the algorithms to the design paradigms they are based on.

(A) P-(ii), Q-(iii), R-(i)  
(B) P-(iii), Q-(i), R-(ii)  
(C) P-(ii), Q-(i), R-(iii)  
(D) P-(i), Q-(ii), R-(iii)

Key: (C)

Exp: Kruskal’s algorithm follows greedy approach in order to find MST of a connected graph. Quick sort follows divide and conquer strategy. Floyd Warshal algorithm is used to find the shortest path between every pair of vertices and it follows dynamic programming strategy.

5. A sender S sends a message \(m\) to receiver R, which is digitally signed by S with its private key. In this scenario, one or more of the following security violations can take place.
   I. S can launch a birthday attack to replace \(m\) with a fraudulent message.
   II. A third party attacker can launch a birthday attack to replace \(m\) with a fraudulent message.
III. R can launch a birthday attack to replace m with a fraudulent message.
Which of the following are possible security violations?
(A) I and II only       (B) I only       (C) II only       (D) II and III only
Key: (B)
Exp: Sender can launch a Birthday Attack to replace with fraudulent message, because he has the signature and he can decrypt the signature by his own public key and gets the hash value. With that same hash value, he can create another message and can be sent instead of original. Hence option(B) is correct.

6. Consider the following grammar.

\[
\begin{align*}
P & \rightarrow xQRS \\
Q & \rightarrow yz|z \\
R & \rightarrow w|\epsilon \\
S & \rightarrow y
\end{align*}
\]
What is FOLLOW (Q) ?
(A) \{R\}   (B) \{w\}   (C) \{w, y\}   (D) \{w, S\}
Key: (C)
Exp: FOLLOW(Q) is FIRST(R) hence 
FIRST(R) = \{w, \epsilon\}
We add “w” in FOLLOW(Q) and for \epsilon we calculate FIRST(S)
FIRST(S) =\{y\}
FOLLOW(Q) is \{w, y\}

7. Consider the language L given by the regular expression \((a+b)^* b(a+b)\) over the alphabet \{a, b\}. The smallest number of states needed in a deterministic finite-state automation (DFA) accepting L is ___________.
Key: (4)
Exp: The regular expression can be described as “All strings over \{a, b\} ending with “ba” or “bb”. The minimal DFA accepting L is having 4 states:

8. Consider a two-level cache hierarchy with L1 and L2 caches. An application incurs 1.4 memory accesses per instruction on average. For this application, the miss rate of L1 cache
0.1, the L2 cache experiences, on average, 7 misses per 1000 instructions. The miss rate of L2 expressed correct to two decimal places is _________.

**Key:** (0.05)

**Exp:**
\[
\text{Number of memory access in 1000 instructions} = 1.4 \times 1000 = 1,400 \\
\therefore \text{Miss Rate} = \frac{7}{1400 \times 0.1} = 0.05
\]

9. Consider the following CPU processes with arrival times (in milliseconds) and length of CPU burst (in milliseconds) as given below:

<table>
<thead>
<tr>
<th>Process</th>
<th>Arrival time</th>
<th>Burst time</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>P2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>P3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>P4</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

If the pre-emptive shortest remaining time first scheduling algorithm is used to schedule the processes, then the average waiting time across all processes is ________ milliseconds.

**Key:** (3)

**Exp:**

<table>
<thead>
<tr>
<th>PID</th>
<th>AT</th>
<th>BT</th>
<th>CT</th>
<th>TAT</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0</td>
<td>7</td>
<td>12</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>P2</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>P3</td>
<td>5</td>
<td>5</td>
<td>17</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>P4</td>
<td>6</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Gantt chart:**

```
P1       P2       P2       P4       P1       P3
0   3   5   6   8   12   17
P1-7  P1-4  P1-4  P1-4  P1-4  P3-5
P2-3  P2-1  P3-5  P3-5
P3-5  P4-2
```

\[ \therefore \text{Average waiting time} = \frac{5+0+7+0}{4} = \frac{12}{4} = 3 \text{ms} \]
10. Threads of a process share
   (A) global variable but not heap.
   (B) heap but not global variables.
   (C) neither global variables nor heap.
   (D) Both heap and global variables.

   **Key:** (D)

   **Exp:** Threads of a process can share all resources except stack and register set.

11. Let $c_1, \ldots, c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where $a_i$ are column vectors in $\mathbb{R}^n$. Consider the set of linear equations $Ax = b$

   where $A = [a_1, \ldots, a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

   (A) a unique solution at $x = J_n$ where $J_n$ denotes a $n$-dimensional vector of all 1
   (B) no solution
   (C) infinitely many solutions
   (D) finitely many solutions

   **Key:** (C)

   **Exp:** Since the scalars are not all zero

   $\therefore$ The column vectors $a_i$ for $i = 1, 2, \ldots, n$ are linearly dependent

   $\Rightarrow |A| = 0$ and $b = \sum_{i=1}^{n} a_i \Rightarrow Ax = b$ has infinitely many solutions.

12. Consider the C code fragment given below.

   ```c
   typedef struct node {
       int data;
       node* next;
   } node;
   void join(node* m, node* n) {
       node* p = n;
       while (p->next != NULL) {
           p = p->next;
       }
       p->next = m;
   }
   ```

   Assuming that $m$ and $n$ point to valid NULL-terminated linked lists, invocation of join will

   (A) append list $m$ to the end of list $n$ for all inputs.
   (B) either cause a null pointer dereference or append list $m$ to the end of list $n$.
   (C) cause a null pointer dereference for all inputs.
   (D) append list $n$ to the end of list $m$ for all inputs.
Key: (B)

Exp: While loop in Join Procedure moves the pointer ‘p’ to the last node of the list “n”. And at the last statement, we are initializing the next of the last node of list n to start of list “m”.
But in some cases it may dereference to null pointer.

13. The n-bit fixed-point representation of an unsigned real number real X uses $f$ bits for the fraction part. Let $i = n - f$. The range of decimal values for X in this representation is

(A) $2^{-i}$ to $2^i$
(B) $2^{-i}$ to $\left(2^i - 2^{-i}\right)$
(C) 0 to $2^i$
(D) 0 to $\left(2^i - 2^{-i}\right)$

Key: (D)

Exp: 
$$i = n - f \quad f$$
Max value = $111.....1(i \text{ times}).111....1(f \text{ times})$
$$= 2^i - 1 + \left(\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^i}\right)$$
$$= 2^i - 1 + \frac{\frac{1}{2} - \frac{1}{2^i}}{\frac{1}{2}}$$
$$= 2^i - 2^{-f}$$
\[\therefore 0 \text{ to } \left(2^i - 2^{-i}\right)\]

14. Consider the following intermediate program in three address code
$$p = a - b$$
$$q = p * c$$
$$p = u * v$$
$$q = p + q$$

Which one of the following corresponds to a static single assignment form of the above code?

(A) $p_i = a - b$
$q_i = p_i * c$
$p_i = u * v$
$q_i = p_i + q_i$

(B) $p_i = a - b$
$q_i = p_i * c$
$p_i = u * v$
$q_i = p_i + q_i$

(C) $p_i = a - b$
$q_i = p_i * c$
$p_i = u * v$
$q_i = p_i + q_i$

(D) $p_i = a - b$
$q_i = p_i * c$
$p_i = u * v$
$q_i = p_i + q_i$

Key: (B)

Exp: 
(a) code violates condition for static single assignment since $p_i$ is initialized twice
(c) $p_2, p_4, \& q_3$ are not initialized anywhere
(d) $q_2 = p + q$ is incorrect code

15. Consider the C struct defined below:
struct data {
    int marks [100];
}
char grade;
int cnumber;
};

struct data student;
The base address of student is available in register R1. The field student.grade can be accessed efficiently using
(A) Post-increment addressing mode. (R1)+
(B) Pre-decrement addressing mode, -(R1)
(C) Register direct addressing mode, R1
(D) Index addressing mode, X(R1), where X is an offset represented in 2’s complement 16-bit representation.

Key: (D)
Exp: Direct access is possible with only index addressing mode.

16. Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls close to terminate the connectional and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK which is received by the client-side TCP. As per the TCP connections state diagram (RFC 793), in which state does the client-side TCP connection wait for the FIN from the server-side TCP?
(A) LAST-ACK    (B) TIME-WAIT    (C) FIN-WAIT-1    (D) FIN-WAIT-2

Key: (D)
Exp: Client* or vice-versa, though requests typically originate at clients.

*or vice-versa, though requests typically originate at clients.

3 syn – sent
Sent connection – request.1
Awaiting acknowledgement.1
Awaiting connection – request.2
Received acknowledgement.1
Received connection – request.2
Sent acknowledgement.2

4 Syn – Received
Received connection – request.1
Sent acknowledgement.1
Sent connection – request.2
Awaiting acknowledgement.2

5 Established
The connection is open.
Data moves both directions.

5 Established
Received acknowledgement.2
The connection is open.
Data moves both directions.

6 Fin – Wait.1
Sent close – request.a
Awaiting acknowledgement.a
Awaiting close – request.b

8 Close – wait
Received close – request.a
Sent acknowledgement.a
When finished sending data,
Will send close – request.b
17. Consider the following context-free grammar over the alphabet \( \Sigma = \{a, b, c\} \) with \( S \) as the start symbol.

\[
S \rightarrow abScT | abcT \\
T \rightarrow bT | b
\]

Which one of the following represents the language generated by the above grammar?

(A) \( \{(ab)^n (cb)^n | n \geq 1\} \)

(B) \( \{(ab)^n cb^{m_1}cb^{m_2}...cb^{m_n} | n, m_1, m_2, ..., m_n \geq 1\} \)

(C) \( \{(ab)^n (cb^n)^n | m, n \geq 1\} \)

(D) \( \{(ab)^n (cb^n)^m | m, n \geq 1\} \)

**Key:** (B)

**Exp:** The given Grammar over \( \Sigma = \{a, b, c\} \) with \( S \) as the start symbol is

\[
S \rightarrow abScT | abcT \\
T \rightarrow bT | b
\]

The minimum length string generated by the grammar is 1:

\( S \rightarrow abcT \rightarrow abc \); hence all variable greater than 1.

Other cases

\( S \rightarrow abScT \rightarrow ab | abScT cT \rightarrow ab abScT cT cT \rightarrow .... \rightarrow (ab)^n (cT)^n \).

Here \( T \) can generate any number of \( b \)'s starting with single \( b \).

Hence The language is \( L = \{(ab)^n cb^{m_1}cb^{m_2}cb^{m_3}cb^{m_4}...cb^{m_n} | m_1, m_2, m_3, m_4, ..., m_n \geq 1\} \)
18. Consider the first-order logic sentence $F: \forall z(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by $F$?

I. $\exists y(\exists x R(x, y))$

II. $\forall y(\exists x R(x, y))$

III. $\forall x(\exists y R(x, y))$

IV. $\neg\exists x(\forall y \neg R(x, y))$

(A) IV only  (B) I and IV only  (C) II only  (D) II and III only

Key: (B)

Exp: $\forall x(\exists y R(x, y)) \Rightarrow \exists y\exists x R(x, y)$

$\exists y\forall x R(x, y) \Rightarrow \forall x\exists y R(x, y)$

$\forall x\exists y R(x, y) \not\Rightarrow \exists y\forall x R(x, y)$

$\neg\exists x(\forall y \neg R(x, y)) \iff \forall x\exists y R(x, y)$

19. When two 8-bit numbers $A_0,...,A_7$ and $B_0,...,B_7$ in 2’s complement representation (with $A_0$ and $B_0$ as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7,...,S_0$ and the carry bits are $C_7,...,C_0$. An overflow is said to have occurred if

(A) the carry bit $C_7$ is 1  
(B) all the carry bits ($C_7,...,C_0$) are 1

(C) $(A_7 B_7 S_7 + A_7 \overline{B_7} S_7)$ is 1  
(D) $(A_7 B_7 S_7 + \overline{A_7} B_7 S_7)$ is 1

Key: (C)

Exp: Overflow flag indicates an overflow condition for a signed operation. Some points to remember in a signed operation:

* MSB is always reserved to indicate sign of the number.

* Negative numbers are represented in 2’s – complement.

* An overflow results in invalid operation.

2’s complement overflow rules:

* If the sum of two positive numbers yields a negative result, the sum has-overflowed.

* If the sum of two negative number yields a positive result, the sum has overflowed.

* Otherwise, the sum has not overflowed.

Overflow for signed numbers occurs when the carry-in into the MSB (most significant bit) is not equal to carry-out. Conveniently, an XOR-operation on these two bits can quickly determine if an overflow condition exists.

Therefore, $(A_7 B_7 S_7 + A_7 \overline{B_7} S_7 + A_7 B_7 S_7 = 1)$ has overflowed.

20. Consider a database that has the relation schema EMP (EmpId, EmpName, and DeptName). An instance of the schema EMP and a SQL query on it are given below.

<table>
<thead>
<tr>
<th>EMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>EmpId</td>
</tr>
<tr>
<td>1.</td>
</tr>
<tr>
<td>2.</td>
</tr>
</tbody>
</table>
3. XYC AA
4. XYD AA
5. XYE AB
6. XYF AB
7. XYG AB
8. XYH AC
9. XYI AC
10. XYJ AC
11. XYK AD
12. XYL AD
13. XYM AE

The output of executing the SQL query is _____.

**Key:** (2.6)

**Exp:**

```
SELECT AVG(EC.Num)
FROM EC
WHERE (DeptName, Num) IN
    (SELECT DeptName, COUNT(EmpId) AS EC DeptName, Num
     FROM EMP
     GROUP BY DeptName)
```

Avg (NUM) = \( \frac{13}{5} = 2.6 \)

21. The following functional dependencies hold true for the relational schema \( R\{V,W,X,Y,Z\} \):
   - \( V \rightarrow W \)
   - \( VW \rightarrow X \)
   - \( Y \rightarrow VX \)
   - \( Y \rightarrow Z \)
Which of the following is irreducible equivalent for this set of functional dependencies?

(A) \( V \rightarrow W \)  
(B) \( V \rightarrow W \)  
(C) \( V \rightarrow W \)  
(D) \( V \rightarrow W \)

\[
\begin{align*}
V & \rightarrow X \\
W & \rightarrow X \\
V & \rightarrow X \\
Y & \rightarrow V \\
Y & \rightarrow V \\
Y & \rightarrow Z \\
Y & \rightarrow Z
\end{align*}
\]

Key: (A)

Exp: \( V \rightarrow W, VW \rightarrow X, Y \rightarrow V, Y \rightarrow V, Y \rightarrow Z \) (\( W \) is extraneous)

\[
\begin{align*}
V & \rightarrow W, V \rightarrow X, Y \rightarrow V, Y \rightarrow X, Y \rightarrow Z \\
\therefore & Y \rightarrow X is redundant \\
\therefore & \{V \rightarrow W, V \rightarrow X, Y \rightarrow V, Y \rightarrow Z\}
\end{align*}
\]

22. Consider the following functions from positive integers to real numbers:

\[
10, \sqrt{n}, n, \log_2 n, \frac{100}{n}
\]

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

(A) \( \log_2 n, \frac{100}{n}, 10, \sqrt{n}, n \)  
(B) \( \frac{100}{n}, \log_2 n, \sqrt{n}, n \)  
(C) \( 10, \frac{100}{n}, \sqrt{n}, \log_2 n, n \)  
(D) \( \frac{100}{n}, \log_2 n, 10, \sqrt{n}, n \)

Key: (B)

Exp: \( \frac{100}{n} < 10 < \log_2 n < \sqrt{n}, n \)

23. Let \( T \) be a tree with 10 vertices. The sum of the degrees of all the vertices in \( T \) is _________.

Key: (18)

Exp: A tree with 10 vertices has 9 edges.

\[
\sum d(v_i) = 2|E| \\
\Rightarrow \sum d(v_i) = 2 \times 9 = 18
\]

24. Let \( T \) be a binary search tree with 15 nodes. The minimum and maximum possible heights of \( T \) are:

Note: The height of a tree with a single node is 0.

(A) 4 and 15 respectively  
(B) 3 and 14 respectively  
(C) 4 and 14 respectively  
(D) 3 and 15 respectively
25. Consider the following C code:

```c
#include <stdio.h>

int * assignval (int *x, int val)  {
    *x = val;
    return x;
}

void main ()  {
    int  * x= malloc (sizeof (int));
    if (NULL = = x) return;
    x = assignval (x,0);
    if(x)   {
        x=(int *) malloc (sizeof (int));
        if (NULL = = x) return;
        x = assignval (x, 10);
    }
    printf("%d\n", *x);
    free (x);
}
```

The code suffers from which one of the following problems:
(A) compiler error as the return of malloc is not typecast appropriately.
(B) compiler error because the comparison should be made as x == NULL and not as shown.
(C) compiles successfully but execution may result in dangling pointer.
(D) compiles successfully but execution may result in memory leak.

Key: (D)

Exp: (A) is wrong. We don’t need to cast the result as void * is automatically and safely promoted to any other pointer type in this case.
(B) It is discarded for obvious reason.
(C) is wrong, because dangling pointer is nothing but the pointer which is pointing to non-existing memory (deallocated or deleted memory) which is not happening here.

(D) is the answer. When you are calling malloc second time, new location is assigned to \( x \) and previous memory location is lost and now we don’t have no reference to that location resulting in memory leak.

Q. No. 26 – 55 Carry Two Marks Each

26. Consider a combination of T and D flip-flops connected as shown below. The output of the D flip-flop is connected to the input of the T flip-flop and the output of the T Flip-flop is connected to the input of the D Flip-flop.

Initially, both \( Q_0 \) and \( Q_1 \) are set to 1 (before the 1\(^{st} \) clock cycle). The outputs

- (A) \( Q_1, Q_0 \) after the 3\(^{rd} \) cycle are 11 and after the 4\(^{th} \) cycle are 00 respectively
- (B) \( Q_1, Q_0 \) after the 3\(^{rd} \) cycle are 11 and after the 4\(^{th} \) cycle are 01 respectively
- (C) \( Q_1, Q_0 \) after the 3\(^{rd} \) cycle are 00 and after the 4\(^{th} \) cycle are 01 respectively
- (D) \( Q_1, Q_0 \) after the 3\(^{rd} \) cycle are 01 and after the 4\(^{th} \) cycle are 01 respectively

Key: (B)

Exp:

<table>
<thead>
<tr>
<th>CLK</th>
<th>( Q_1 )</th>
<th>( Q_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

After 3\(^{rd} \) clock pulse: 11
After 4\(^{th} \) clock pulse: 01

27. The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is

Key: (271)

Exp:

\[ D_3 = \{ \text{integers between 1 to 500 divisible by 3} \} \]

\[ D_5 = \{ \text{integers between 1 to 500 divisible by 5} \} \]

\[ D_7 = \{ \text{integers between 1 to 500 divisible by 7} \} \]
To find number of integers between 1 to 500 that are divisible by 3 or 5 or 7 is to find 
\[|D_3 \cup D_5 \cup D_7|\]
\[= \left|D_3\right| + \left|D_5\right| + \left|D_7\right| - \left|D_3 \cap D_5\right| - \left|D_3 \cap D_7\right| - \left|D_5 \cap D_7\right| + \left|D_3 \cap D_5 \cap D_7\right|\]
\[= \left(\frac{500}{3}\right) + \left(\frac{500}{5}\right) + \left(\frac{500}{7}\right) - \left(\frac{500}{15}\right) - \left(\frac{500}{21}\right) - \left(\frac{500}{35}\right) + \left(\frac{500}{105}\right)\]
\[= (166 + 100 + 71) - (33 + 23 + 14) + 4\]
\[= 337 - 70 + 4 = 271\]

28. Consider a RISC machine where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions use PC-relative addressing mode with Offset specified in bytes to the target location of the branch instruction. Further the Offset is always with respect to the address of the next instruction in the program sequence. Consider the following instruction sequence.

<table>
<thead>
<tr>
<th>Instr. No.</th>
<th>Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>i:</td>
<td>add R2, R3, R4</td>
</tr>
<tr>
<td>i +1:</td>
<td>sub R5, R6, R7</td>
</tr>
<tr>
<td>i + 2:</td>
<td>cmp R1, R9, R10</td>
</tr>
<tr>
<td>i + 3:</td>
<td>beq R1, Offset</td>
</tr>
</tbody>
</table>

If the target of the branch instruction is i, then the decimal value of the Offset is \(-16\).

Key: \((-16)\)

Exp:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I₀</td>
<td>0 – 3</td>
</tr>
<tr>
<td>I₁</td>
<td>4 – 7</td>
</tr>
<tr>
<td>I₂</td>
<td>8 – 11</td>
</tr>
<tr>
<td>I₃</td>
<td>12 – 15</td>
</tr>
<tr>
<td>I₄</td>
<td>16 –</td>
</tr>
</tbody>
</table>

I₁ is the branch instruction & I₀ is the target.

0 = 16+ relative value

∴ relative value = -16

29. Consider the C functions foo and bar given below:

```c
int foo (int val ) {
    int x = 0;
    while (val > 0) {
        x = x + foo ( val --);
    }
    return val ;
}
```

```c
int bar (int val ) {
    int x = 0;
    while (val > 0) {
        x = x + bar (val - 1) ;
    }
    return val ;
}
```
Invocations of foo (3) and bar (3) will result in:
(A) Return of 6 and 6 respectively.
(B) Infinite loop and abnormal termination respectively.
(C) Abnormal termination and infinite loop respectively.
(D) Both terminating abnormally

Key: (B)

Exp:
Foo (3) calls foo (3) which in turn calls foo(3). This goes on infinite number of times which causes memory overflow and causes abnormal termination.
Bar(3) → bar (2) → bar (1) → bar (0) (return 0) from here onwards bar (1) will call bar (0) and bar (0) will return 0 to bar (1) & this goes on forever without causing memory overflow.

30. In a RSA cryptosystem a participant A uses two prime numbers p = 13 and q = 17 to generate her public and private keys. If the public key of A is 35. Then the private key of A is __________.

Key: (11)

Exp:

<table>
<thead>
<tr>
<th>Given Data</th>
<th>As per RSA Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>p=13</td>
<td><strong>Step 1:</strong> Calculate ( n = p \times q = 13 \times 17 = 221 )</td>
</tr>
<tr>
<td>q=17</td>
<td><strong>Step 2:</strong> Calculate ( \phi(n) = (p-1)(q-1) = (12)(16) = 192 )</td>
</tr>
<tr>
<td>e=35</td>
<td><strong>Step 3:</strong> de \mod \phi(n) = 1 (or) de \mod \phi(n) = 1</td>
</tr>
<tr>
<td>d=?</td>
<td>( d \times 35 \mod 192 = 1 \Rightarrow d = 11 )</td>
</tr>
</tbody>
</table>

31. Let A be an array of 31 numbers consisting of sequence of 0’s followed by a sequence of 1’s. The problem is to find the smallest index \( i \) that \( A[i] \) is 1 by probing the minimum numbers of locations in A. The worst case number of probes performed by an optimal algorithm is __________.

Key: (5)

Exp:
In the given array the elements are 0’s followed by 1’s, which means array is already sorted.

So we can apply binary search. At each stage, we compare \( A \left[ \frac{\text{low} + \text{high}}{2} \right] \).

[Assuming ‘A’ is an array of 31 elements] with ‘1’ and if it is 1 we check the left part recursively and if it is ‘0’ we check the right part of the array recursively, which takes \( \log_2 31 \) comparisons in the worst case.

32. If G is grammar with productions

\[
S \rightarrow SaS | aSb | bSa | S | e
\]

where S is the start variable, then which one of the following is not generated by G?
Given grammar generates all strings where \( n(a) \geq n(b) \)

33. The value of \( \lim_{x \to 0} \frac{x^7 - 2x^3 + 1}{x^3 - 3x^2 + 2} \)

(A) is 0  
(B) is -1  
(C) is 1  
(D) does not exist

Key: (C)

Exp: 
\[ \lim_{x \to 0} \frac{x^7 - 2x^3 + 1}{x^3 - 3x^2 + 2} = \lim_{x \to 0} \frac{7x^6 - 10x^4}{3x^2 - 6x} = 1. \] (Using L–Hospital’s rule)

34. Instructions execution in a processor is divided into 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Execute (EX), and Write Back (WB). These stages take 5, 4, 20, 10 and 3 nanoseconds (ns) respectively. A pipelined implementation of the processor requires buffering between each pair of consecutive stages with a delay of 2ns. Two pipelined implementations of the processor are contemplated.

(i) a naïve pipeline implementation (NP) with 5 stages and
(ii) an efficient pipeline (EP) where the OF stage is divided into stages OF1 and OF2 with execution times of 12 ns and 8 ns respectively.

The speedup (correct to two decimals places) achieved by EP over NP in executing 20 independent instructions with no hazards is ___________.

Key: (1.508)

Exp: 

Given, 
For Navie pipeline (NP)
Number of stages (k) = 5
\[ T_p = \max \text{ (stage delay + buffer delay)} \]
\[ T_p = \max (7, 6, 22, 12, 5) = 22 \text{ n sec.} \]
Number of instructions (n) = 20
So, erection time for navie pipeline
\[ ET_{np} = (k + n - 1) \times T_p = (5 + 20 - 1) \times 22 = 528 \text{ n sec} \]

Now, for efficient pipeline
\[ k = 6, n = 20, T_p = 14 \text{ n sec.} \]
\[ E_{ep} = (k + n - 1) \times T_p = (6 + 20 - 1) \times 14 = 350 \text{ nsec.} \]

Therefore, Speedup \((s) = \frac{ET_{np}}{E_{ep}} = \frac{528}{350} \]
Speedup \((s) = 1.508 \)
35. Consider a database that has the relation schemas EMP(EmpId, EmpName, DeptId). And DEPT(DeptName, DeptId). Note that the DeptId can be permitted to be NULL in the relation EMP. Consider the following queries on the database expressed in tuple relational calculus.

(I) \( \{ t \mid \exists u \in EMP(t[\text{EmpName}]=u[\text{EmpName}] \land \forall v \in DEPT(t[\text{DeptId}] \neq v[\text{DeptId}]) \} \)  

(II) \( \{ t \mid \exists u \in EMP(t[\text{EmpName}]=u[\text{EmpName}] \land \exists v \in DEPT(t[\text{DeptId}] \neq v[\text{DeptId}]) \} \)  

(III) \( \{ t \mid \exists u \in EMP(t[\text{EmpName}]=u[\text{EmpName}] \land \exists v \in DEPT(t[\text{DeptId}] \neq v[\text{DeptId}]) \} \)  

Which of the above queries are safe?

(A) (I) and (II) only  
(B) (I) and (III) only  
(C) (II) and (III) only  
(D) (I), (II) and (III)

Key: (D)

Exp: Query which generates infinite number of tuples is called unsafe query. In the given question all the given queries generate finite number of tuples.

36. Recall that Belady’s anomaly is that the pages-fault rate may increase as the number of allocated frames increases. Now consider the following statements:

S1: Random page replacement algorithm (where a page chosen at random is replaced) suffers from Belady’s anomaly  
S2: LRU page replacement algorithm suffers from Belady’s anomaly

Which of the following is CORRECT?

(A) S1 is true, S2 is true  
(B) S1 is true, S2 is false  
(C) S1 is false, S2 is true  
(D) S1 is false, S2 is false

Key: (B)

Exp: Statement 1 is “TRUE”. Because there can be a case when page selected to be replaced is by FIFO policy.  
Statement 2 is “FALSE”. Because LRU page replacement algorithm does not suffers from Belady’s Anomaly. Only FIFO page replacement algorithm suffers from Belady’s Anomaly.

37. The output of executing the following C program is ________.

```c
#include <stdio.h>

int total (int v) {
    while (v) {
        count += v & 1;
        v >>= 1;
    }
    return count;
}

void main ( ) {
    static int x = 0;
    int i = 5;
    for (; i> 0; i--) {
        
```
38. Consider the following C program.
#include <stdio.h>
#include <string.h>

void printlength (char *s, char *t) {
  unsigned int c = 0;
  int len = ((strlen(s) – strlen(t)) > c) ? strlen(s): strlen(t);
  printf (“%d
”, len);
}

void main (  ) {
  char  *x = “abc”;
  char  *y = “defgh”;
  printlength (x,y);
}

Recall that strlen is defined in string.h as returning a value of type size_t, which is an unsigned int. The output of the program is ___________.

Key: (3)
Exp: x is pointer of string “abc” which is length 3.
S is pointer, that pointed x.
y is pointer of string “defgh” which is length 5.
t is pointer that pointed y.

Now, ((strlen(s) – strlen(t)) > c) is ((3 – 5) > 0) is returns true, since (3 – 5 = -2) is non-zero value so, ternary operator “?” is returned strlen(s) which is 3 and assigned to len because ternary- operator returns first if condition is true else second. Therefore, final value is printed 3.

39. Consider the following languages over the alphabet \( \sum = \{a,b,c\} \)
Let \( L_1 = \{ a^n b^n c^m \mid m,n \geq 0 \} \) and \( L_2 = \{ a^n b^n c^n \mid m,n \geq 0 \} \)

Which of the following are context-free languages?

I. \( L_1 \cup L_2 \) 
II. \( L_1 \cap L_2 \)

(A) I only  (B) II only  (C) I and II  (D) Neither I nor II

Key: (A)

Exp:
The language given over alphabets \( \Sigma = \{ a, b, c \} \) as \( L_1 = \{ a^n b^n c^m \mid n,m \geq 0 \} \) and \( L_2 = \{ a^n b^n c^n \mid n,m \geq 0 \} \).

\( L_1 \cup L_2 = \{ a^n b^n c^m \mid n=0 \text{ or } m=0 \} \) is a context free language. The context free grammar is:

\[
S \rightarrow AB|CD \\
A \rightarrow aAb | \epsilon \\
B \rightarrow cB | \epsilon \\
C \rightarrow aC | \epsilon \\
D \rightarrow bSc | \epsilon
\]

\( L_1 \cap L_2 = \{ a^n b^n c^m \mid n=m, m=0 \} \) or \( \{ a^n b^n c^n \mid n \geq 0 \} \) is a non context free language.

40. Consider a 2-way set associative cache with 256 blocks and uses LRU replacement. Initially the cache is empty. Conflict misses are those misses which occur due the contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks,

\( (0, 128, 256, 128, 0, 128, 256, 128, 1, 129, 257, 129, 1, 129, 257, 129, 1, 129, 257, 129) \)

is repeated 10 times. The number of conflict misses experienced by the cache is ___________.

Key: (76)

Exp:
A miss is not considered a conflict miss if the block is accessed for the first time.

1st round: (2+2) misses 
2nd round: (4+4) misses

.: Total = 4+(8×9) = 76 conflict misses

41. Let \( u \) and \( v \) be two vectors in \( \mathbb{R}^2 \) whose Euclidean norms satisfy \( \|u\| = 2\|v\| \). What is the value of \( \alpha \) such that \( w = u + \alpha v \) bisects the angle between \( u \) and \( v \)?

(A) 2  (B) 1/2  (C) 1  (D) -1/2

Key: (A)

Exp:
Let \( u = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \) and \( v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)

\[ \Rightarrow \|u\| = z \|v\| \text{ and } w = \begin{pmatrix} 2 \\ \alpha \end{pmatrix} \]

Now \( \cos(u, w) = \cos(v, w) \)

\[ \Rightarrow \frac{4}{(2)\sqrt{\alpha^2 + 4}} = \frac{\alpha}{(1)\sqrt{\alpha^2 + 4}} \Rightarrow \alpha = 2 \]
42. Consider the following grammar:

```plaintext
stmt → if expr then else expr; stmt
expr → term rel op term | term
term → id | number
if → a | b | c
number → [0 – 9]
```

where `rel op` is a relational operator (e.g., <, >, ....) --- `O` refers to the empty statement, and `if`, `then`, `else` are terminals.

Consider a program `P` following the above grammar containing ten `if` terminals. The number of control flows paths in `P` is __________. For example the program

if `e_1` then `e_2` else `e_3`

has 2 controls flow paths `e_1` → `e_2` and `e_1` → `e_3`

**Key:** (1024)

**Exp:** For 2 “if statements”, `2^2=4` control flow paths are possible:

```
  e_1
     |
   /  \\
  if  if
    |   |
e_2  e_3
```

So for 10 “If statements”, `2^{10}` control flow paths will be there.

43. In a database system, unique time stamps are assigned to each transaction using Lamport’s logical clock. Let `TS(T_1)` and `TS(T_2)` be the timestamps of transactions `T_1` and `T_2` respectively. Besides, `T_1` holds a lock on the resource `R`, and `T_2` has requested a conflicting lock on the same resource `R`. The following algorithm is used to prevent deadlocks in the database system assuming that a killed transaction is restarted with the same timestamp.

```plaintext
if `TS(T_i) < TS(T_j)` then
  `T_i` is killed
else `T_j` waits.
```

Assume any transactions that is not killed terminates eventually. Which of the following is TRUE about the database system that uses the above algorithm to prevent deadlocks?

(A) The database system is both deadlock-free and starvation-free.
(B) The database system is deadlock-free, but not starvation-free.
(C) The database system is starvation-free but not deadlock-free.
(D) The database system is neither deadlock-free nor starvation-free.

**Key:** (A)

**Exp:** Elder kills younger and youngers waits on elder. So both are not waiting for each other. Hence no deadlock and there won’t be any starvation as well because the transaction who got killed will be starting with same time stamp.
44. Let A and B be infinite alphabets and let # be a symbol outside both A and B. Let $f$ be a total functional from $A^*$ to $B^*$. We say $f$ is computable if there exists a Turning machine $M$ which given an input $x$ in $A^*$, always halts with $f(x)$ on its tape. Let $L_f$ denote the language $\{x \# f(x) | x \in A^*\}$. Which of the following statements is true:

(A) $f$ is computable if and only if $L_f$ is recursive.
(B) $f$ is computable if and only if $L_f$ is recursively enumerable.
(C) If $f$ is computable then $L_f$ is recursive, but not conversely.
(D) If $f$ is computable then $L_f$ is recursively enumerable, but not conversely.

**Key:** (A)

**Exp:** A TM is recursive iff it halts for every input string (either in accept or reject state). Here, a computable function is defined in a similar way.

45. Consider the expression $(a-1)^{(\frac{(b+c)}{3})+d)}$. Let $X$ be the minimum number of registers required by an optimal code generation (without any register spill) algorithm for a load/store architecture in which (i) only loads and store instructions can have memory operands and (ii) arithmetic instructions can have only register or immediate operands. The value of $X$ is _________.

**Key:** (2)

**Exp:**

The given expression is $(a-1)^{(\frac{(b+c)}{3})+d)}$

The optimal generated code is:

- LOAD R1 b  \( R_1 \leftarrow b \)
- LOAD R2 c  \( R_2 \leftarrow c \)
- ADD R1 R2  \( R_1 \leftarrow R_1 + R_2 \)
- DIV R1 3  \( R_1 \leftarrow R_1 / 3 \)
- LOAD R2 d  \( R_2 \leftarrow d \)
- ADD R1 R2  \( R_1 \leftarrow R_1 + R_2 \)
- LOAD R2 a  \( R_2 \leftarrow a \)
- SUB R2 1  \( R_2 \leftarrow R_2 - 1 \)
- MUL R2 R1  \( R_2 \leftarrow R_2 \times R_1 \)
46. Let G = (V, E) be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:
(I) Minimum spanning tree of G is always unique.
(II) Shortest path between any two vertices of G is always unique.
Which of the above statements is/are necessarily true?
(A) (I) only  
(B) (II) only  
(C) Both (I) and (II)  
(D) Neither (I) nor (II)  

Key: (A)

Exp: Shortest path from B to C are two B-A-C and B-C both of weight '3'

47. A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are non-reentrant, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:
(A) x = 1, y = 2  
(B) x = 2, y = 1  
(C) x = 2, y = 2  
(D) x = 1, y = 1  

Key: (C)

Exp: As per given question, there ‘x’ number of threads and ‘y’ number of locks for ensuring mutual exclusion while operating on shared memory locations

Option (A): x=1; y=2
Means that 1 thread and 2 locks clearly showing that no deadlock situation

Option (B): x=2; y=1
Means that 2 threads and 1 lock → No deadlock situation

After usage of lock by 1 thread, it can release that lock and then 2nd thread can be used that lock. So no deadlock

Option(C):x=2; y=2
Means that 2 threads and 2 locks → Deadlock can arise
Both threads can hold 1 lock and can wait for release of another lock

Option(D) x=1; y=1
Means that 1 thread and 1 lock → No deadlock situation

Hence Option(C) is correct.

48. The values of parameters for the Stop-and-Wait ARQ protocol are as given below:
Bit rate of the transmission channel = 1Mbps
Propagation delay from sender to receiver = 0.75 ms
Time to process a frame = 0.25ms
Number of bytes in the information frame = 1980
Number of bytes in the acknowledge frame = 20
Number of overhead bytes in the information frame = 20
Assume that there are no transmission errors. Then the transmission efficiency (expressed in percentage) of the Stop-and-Wait ARQ protocol for the above parameters is ________ (correct to 2 decimal places)

**Key:** (89.33)

**Exp:**
Given Data:
- B = 1Mbps
- \( T_{\text{proc}} = 0.25 \text{ms} \)
- \( T_p = 0.75 \text{ ms} \)
- L = 1980 Bytes
- \( L_{\text{OH}} = 20 \) Bytes
- \( L_A = 20 \) Bytes

Efficiency \( \eta = ? \)

(i) \( T_s = \frac{L}{B} = \frac{(1980 + 20) \times 8 \times 10^3}{10^6} = 16 \text{ ms} \)

(ii) \( T_{\text{ACK}} = \frac{L_A}{B} = \frac{20 \times 8 \times 10^3}{10^6} = 0.16 \text{ ms} \)

In stop-and-wait ARQ, efficiency

\[ \eta = \frac{T_s}{T_s + T_{\text{ACK}} + 2T_p + T_{\text{proc}}} = \frac{16}{17.91} \approx 0.8933 = 89.33\% \]

49. A computer network uses polynomials over \( GF(2) \) for error checking with 8 bits as information bits and uses \( x^3 + x + 1 \) as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as

(A) 01011011010  (B) 01011011011  (C) 01011011101  (D) 01011011100

**Key:** (C)

**Exp:**
Given generator polynomial\( G(x) = x^3 + x + 1 \)\Rightarrow 1011

message \( m(x) = 01011011 \)

\[
\begin{array}{c}
01011011 \\
0000 \\
0000 \\
0000 \\
0011 \\
0000 \\
0110 \\
0000 \\
1100 \\
1011 \\
1110 \\
1011 \\
1011 \\
\end{array}
\]

\[\text{01000011} \]

\[\text{1011} \]

\[\text{0101} \]

\[\text{1011} \]

\[\text{0110} \]

\[\text{1011} \]

\[\text{1110} \]

\[\text{1011} \]

\[\text{1011} \]
50. Let p, q, and r be propositions and the expression \((p \rightarrow q) \rightarrow r\) be a contradiction. Then, the expression \((r \rightarrow p) \rightarrow q\) is

(A) a tautology  
(B) a contradiction  
(C) always TRUE when p is FALSE  
(D) always TRUE when q is TRUE

Key:  
(D)

Exp:  
\[(p \rightarrow q) \rightarrow r\]  
is contradiction only when

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

And now for the above combination, the expression \((r \rightarrow p) \rightarrow q\) is always true when q is true. When q is false in the above combination (third one) \((r \rightarrow p) \rightarrow q\) will be false.

51. A cache memory unit with capacity of N words and block size of B words is to be designed. If it is designed as a direct mapped cache, the length of the TAG field is 10 bits. If the cache unit is now designed as a 16-way set-associative cache, the length of the TAG field is _____ bits.

Key:  
(14)

Exp:  
Total bits = \(10 + \log_2 \left( \frac{N}{B} \right) + \log_2 (\text{Offset})\)

\[10 + \log_2 (N) = \log_2 \left( \frac{N}{16} \right) + T\]

where T is the required length of TAG field

∴ T = 14

52. Consider the following two functions.

```c
void fun1 (int  n) {   
    if (n == 0) return;    
    printf (“%d”, n);    
    fun2 (n - 2);    
    printf (“%d”, n);    
}

void fun2 (int  n) {   
    if (n == 0) return;    
    printf (“%d”, n);    
    fun1(++n) ;    
    printf (“%d”, n);    
}
```

The output printed when fun1(5) is called is

(A) 53423122233445  
(B) 53423120112233  
(C) 53423122132435  
(D) 53423120213243

Key:  
(A)

Exp:  
In this the fun1() is calling fun2() after printing value and after returning from fun2(),it prints the same value. In the fun2() also the same thing happens So by looking options we can judge the correct sequence of output.
53. Consider a database that has the relation schema CR (StudentName, CourseName). An instance of the schema CR is as given below.

<table>
<thead>
<tr>
<th>Student Name</th>
<th>Course Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>CA</td>
</tr>
<tr>
<td>SA</td>
<td>CB</td>
</tr>
<tr>
<td>SA</td>
<td>CC</td>
</tr>
<tr>
<td>SB</td>
<td>CB</td>
</tr>
<tr>
<td>SB</td>
<td>CC</td>
</tr>
<tr>
<td>SC</td>
<td>CA</td>
</tr>
<tr>
<td>SC</td>
<td>CB</td>
</tr>
<tr>
<td>SC</td>
<td>CC</td>
</tr>
<tr>
<td>SD</td>
<td>CA</td>
</tr>
<tr>
<td>SD</td>
<td>CB</td>
</tr>
<tr>
<td>SD</td>
<td>CC</td>
</tr>
<tr>
<td>SD</td>
<td>CD</td>
</tr>
<tr>
<td>SE</td>
<td>CD</td>
</tr>
<tr>
<td>SE</td>
<td>CA</td>
</tr>
<tr>
<td>SE</td>
<td>CB</td>
</tr>
<tr>
<td>SF</td>
<td>CA</td>
</tr>
<tr>
<td>SF</td>
<td>CB</td>
</tr>
<tr>
<td>SF</td>
<td>CC</td>
</tr>
</tbody>
</table>

The following query is made on the database.

T1 ← π_{CourseName} (σ_{StudentName='SA'} (CR))
T2 ← CR ÷ T1

The number of rows in T2 is ____________.

Key: (4)

Exp: T1 = \{CA, CB, CC\}
T2 = \{SA, SC, SD, SF\}

54. Let A be an n x n real valued square symmetric matrix of rank 2 with \( \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} = 50 \). Consider the following statements.

(I) One eigen value must be in [-5, 5]
(II) The eigen value with the largest magnitude must be strictly greater than 5.
Which of the above statements about eigen values of A is/are necessarily CORRECT?
(A) Both (I) and (II)  (B) (I) only
(C) (II) only  (D) Neither (I) nor (II)

Key: (B)

Exp: \( \rho(A) < n \Rightarrow |A| = 0 \Rightarrow \) one eigen value must be '0' \( \in [-5,5] \)
\[ \therefore (I) \text{ is true} \]

Let \( A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow \sum_{i=1}^{3} \sum_{j=1}^{3} A_{ij}^2 = 50 \text{ and } \rho(A) = 2 \]

but eigen values of A are 0, -5, 5
\[ \therefore \text{The eigen value with the largest magnitude is not greater than 5} \]

For \( n=2 \) and Let \( A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \Rightarrow \text{eigen values= 5,5} \)
\[ \therefore \text{One eigen value must be in } [-5,5] \text{ and largest eigen value magnitude is not greater than 5} \]
\[ \therefore \text{(II) is false} \]

55. Consider the context-free grammars over the alphabet \{a,b,c\} given below. S and T are non-terminals
\[ G_1 : S \rightarrow aSb | T, T \rightarrow cT \]
\[ G_2 : S \rightarrow bSa | T, T \rightarrow cT \]
The language \( L(G_1) \cap L(G_2) \) is

(A) Finite.  (B) Not finite but regular.
(C) Context-free but not regular.  (D) Recursive but not context-free.

Key: (B)

Exp: The Context free grammar given over alphabets \( \Sigma = \{a, b, c\} \) with S and T as non terminals are:
\[ G_1 : S \rightarrow aSb | T, T \rightarrow cT | \epsilon \]
\[ G_2 : S \rightarrow bSa | T, T \rightarrow cT | \epsilon \]
Lets \( L(G_1) \) is the language for grammar \( G_1 \) and \( L(G_2) \) is the language for grammar \( G_2 \)
\[ L(G_1) = \{ a^m c^n b \mid n, m \geq 0 \} \]
\[ L(G_2) = \{ b^m c^n a \mid n, m \geq 0 \} \]
\[ L_1 \cap L_2 = \{ c^m \mid m \geq 0 \}; \text{ which is infinite and regular} \]
General Aptitude

Q. No. 1 - 10 Carry One Mark Each

1. Research in the workplace reveals that people work for many reason ___________.
   (A) money beside   (B) beside money   (C) money besides   (D) besides money
   Key: (D)

2. After Rajendra chola returned from his voyage to Indonesia, he _______ to visit the temple in Thanjavur.
   (A) was wishing   (B) is wishing   (C) wished   (D) had wished
   Key: (C)

3. Rahul Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali. Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other ?
   (A) Rahul and Murali   (B) Srinivas and Arul
   (C) Srinivas and Murali   (D) Srinivas and Rahul
   Key: (C)
   Exp:

4. Find the smallest number y such that \( y \times 162 \) is a perfect cube.
   (A) 24   (B) 27   (C) 32   (D) 36
   Key: (D)
   Exp:

5. The probability that a k-digit number does NOT contain the digits 0,5,or 9 is
   (A) 0.3^k   (B) 0.6^k   (C) 0.7^k   (D) 0.9^k
   Key: (C)
   Exp:

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Each digit can be filled in 7 ways as 0, 5 and 9 are not allowed. So each of these places can be filled by 1, 2, 3, 4, 6, 7, 8.

So required probability is \( \left( \frac{7}{10} \right)^k \) or \( 0.7^k \).

Q. No. 6 – 10 Carry Two Marks Each

6. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25m intervals in this plot. If in a flood, the water level rises to 525m, which of villages P, Q, R, S, T get submerged?

(A) P, Q   (B) P, Q, T   (C) R, S, T   (D) Q, R, S

Key: (C)

Exp: The given contour is a hill station, the peak point of this hill station is P, it is under a contour of 550. At floods, the water level is 525m. So the village of R, S and T are under a contour of 500. Therefore these villages are submerged.

7. “The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history”

Which of the following statements best reflects the author’s opinion?

(A) Nationalists are highly imaginative.
(B) History is viewed through the filter of nationalism.
(C) Our colonial past never happened.
(D) Nationalism has to be both adequately and properly imagined.

Key: (B)

8. The expression \( \frac{(x + y) - |x - y|}{2} \) is equal to

(A) the maximum of x and y       (B) the minimum of x and y
(C) 1                               (D) None of the above

Key: (B)
Exp: If \( x > y \); then \(|x - y| = x - y\)
\[ \text{Exp} = \frac{x + y - (x - y)}{2} = y_{\text{min}} \]
If \( x < y \); then \(|x - y| = -(x - y) = y - x\)
\[ \text{Exp} = \frac{x + y - (y - x)}{2} = x_{\text{min}} \]
\[
\therefore \text{The expression } \frac{x + y - |x - y|}{2} \text{ is equal to minimum of } x \& y
\]

9. Six people are seated around a circular table. There are at least two men and two women. There are at least three right-handed persons. Every woman has a left-handed person to her immediate right. None of the women are right-handed. The number of women at the table is
(A) 2 (B) 3 (C) 4 (D) Cannot be determined

Key: (A)

Exp: Out of six people, 3 place definitely occupied by right handed people as at least 2 women are there so these two will sit adjacently. Now as only one seat is left it will be occupied by a left handed man because on right side of this seat is sitting an right handed man.

Therefore, answer should be 2 women.

10. Arun, Gulab, Neel and Shweta must choose one shirt each from a pile of four shirts coloured red, pink, blue and white respectively. Arun dislikes the colour red and Shweta dislikes the colour white. Gulab and Neel like all the colours. In how many different ways can they choose the shirts so that no one has a shirt with a colour he or she dislikes?
(A) 21 (B) 18 (C) 16 (D) 14

Key: (D)

Exp: As there are 4 people A,G,N,S and 4 colours so without any restriction total ways have to be \(4 \times 4 = 16\)
Now, Arun \(\rightarrow\) dislikes Red and
Shweta \(\rightarrow\) dislikes white
So 16-2=14 ways