

**Electronics and Communication Engineering**

**Q. No. 1 to 25 Carry One Mark Each**

1. The clock frequency of an 8085 microprocessor is 5 MHz. If the time required to execute an instruction is 1.4 μs, then the number of T-states needed for executing the instruction is

- (A) 1                                      (B) 6                                      (C) 7                                      (D) 8

**Key: (C)**

**Exp:**  $f_{\text{clock}} = 5\text{MHz}; T_{\text{clock}} = 0.2 \times 10^{-6} \text{ sec}$

$T_{\text{execution}} = 1.4\mu\text{s}$

No. of T – state required =  $\frac{1.4}{0.2} = 7$

2. Consider a single input single output discrete-time system with  $x[n]$  as input and  $y[n]$  as output, where the two are related as

$$y[n] = \begin{cases} n|x[n]| & \text{for } 0 \leq n \leq 10 \\ x[n] - x[n-1] & \text{otherwise} \end{cases}$$

Which one of the following statements is true about the system?

- (A) It is causal and stable                      (B) It is causal but not stable  
(C) It is not causal but stable                      (D) It is neither causal nor stable

**Key: (A)**

**Exp:** For an input-output relation if the present output depends on present and past input values then the given system is “Causal”.

For the given relation,

$$y[n] = \begin{cases} n|x[n]| & 0 \leq n \leq 10 \\ x[n] - x[n-1] & \text{otherwise} \end{cases}$$

For  $n$  ranging from 0 to 10 present output depends on present input only.

At all other points present output depends on present and past input values.

Thus the system is “Causal”.

**Stability**

If  $x[n]$  is bounded for the given finite range of  $n$  i.e.  $0 \leq n \leq 10$   $y[n]$  is also bounded.

Similarly  $x[n] - x[n-1]$  is also bounded at all other values of  $n$

Thus the system is “stable”.

3. Consider the following statement about the linear dependence of the real valued functions  $y_1 = 1, y_2 = x$  and  $y_3 = x^2$ , over the field of real numbers.

- I.  $y_1, y_2$  and  $y_3$  are linearly independent on  $-1 \leq x \leq 0$   
 II.  $y_1, y_2$  and  $y_3$  are linearly dependent on  $0 \leq x \leq 1$   
 III.  $y_1, y_2$  and  $y_3$  are linearly independent on  $0 \leq x \leq 1$   
 IV.  $y_1, y_2$  and  $y_3$  are linearly dependent on  $-1 \leq x \leq 0$

Which one among the following is correct?

- (A) Both I and II are true (B) Both I and III are true  
(C) Both II and IV are true (D) Both III and IV are true

**Key: (B)**

**Exp:**  $y_1 = 1, y_2 = x, y_3 = x^2$

$$\text{Consider } \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2 \begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix} = 2 \neq 0$$

$\Rightarrow y_1, y_2, y_3$  are linearly independent  $\nexists x$

4. Consider the  $5 \times 5$  matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigen value. Then the real eigen value of A is

- (A) -2.5 (B) 0 (C) 15 (D) 25

**Key: (C)**

**Exp:**  $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$

For eigen values  $(\lambda), |A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 + R_2 + R_3 + R_4 + R_5$

$$\Rightarrow \begin{vmatrix} 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (15 - \lambda) \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow 15 - \lambda = 0$$

$$\Rightarrow \lambda = 15$$

5. The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance is  $V(\ell) = e^{-\gamma\ell + j\omega t}$  volts, Where  $\ell$  is the distance along the length of the cable in meters.  $\gamma = (0.1 + j40)\text{m}^{-1}$  is the complex propagation constant, and  $\omega = 2\pi \times 10^9$  rad/s is the angular frequency. The absolute value of the attenuation in the cable in dB/meter is \_\_\_\_\_.

**Key:** (0.85 to 0.88)

**Exp:** Given  $\gamma = (0.1 + j40)\text{m}^{-1}$

Here  $\alpha = 0.1 \frac{\omega_p}{\text{m}}$

WE know that,  $1 \frac{\omega_p}{\text{m}} = 8.686 \frac{\text{dB}}{\text{m}} \Rightarrow 0.1 \frac{\omega_p}{\text{m}} = 0.8686 \frac{\text{dB}}{\text{m}}$

6. A bar of Gallium Arsenide (GaAs) is doped with Silicon such that the Silicon atoms occupy Gallium and Arsenic sites in the GaAs crystal. Which one of the following statement is true?  
 (A) Silicon atoms act as p-type dopants in Arsenic sites and n-type dopants in Gallium sites  
 (B) Silicon atoms act as n-type dopants in Arsenic sites and p-type dopants in Gallium sites  
 (C) Silicon atoms act as p-type dopants in Arsenic as well as Gallium sites  
 (D) Silicon atoms act as n-type dopants in Arsenic as well as Gallium sites

**Key:** (A)

**Exp:** Silicon atoms act as P- type dopants in Arsenic sites and n- type dopants in Gallium sites.

7. The rank of the matrix  $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$  is

(A) 0

(B) 1

(C) 2

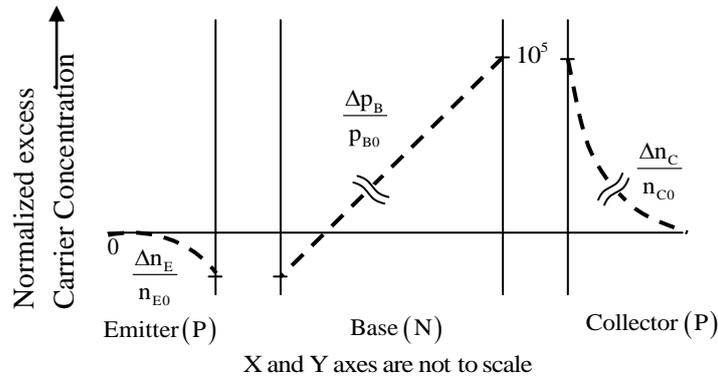
(D) 3

**Key:** (C)

**Exp:**  $|M| = \begin{vmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{vmatrix} = 5(0-12) - 10(6-6) + 10(6-0) = -60 - 0 + 60 = 0$

But a  $2 \times 2$  minor,  $\begin{vmatrix} 5 & 10 \\ 1 & 0 \end{vmatrix} = 0 - 10 = -10 \neq 0 \Rightarrow \text{Rank} = 2$

8. For a narrow base PNP BJT, the excess minority carrier concentration ( $\Delta n_E$  for emitter,  $\Delta p_B$  for base,  $\Delta n_C$  for collector) normalized to equilibrium minority carrier concentration ( $n_{E0}$  for emitter,  $p_{B0}$  for base,  $n_{C0}$  for collector) in the quasi-neutral emitter, base and collector regions are shown below. Which one of the following biasing modes is the transistor operating in?



- (A) Forward active    (B) Saturation    (C) Inverse active    (D) Cutoff

**Key:** (C)

**Exp:** As per the change carrier profile, base – to – emitter junction is reverse bias and base to collector junction is forward bias, so it works in Inverse active.

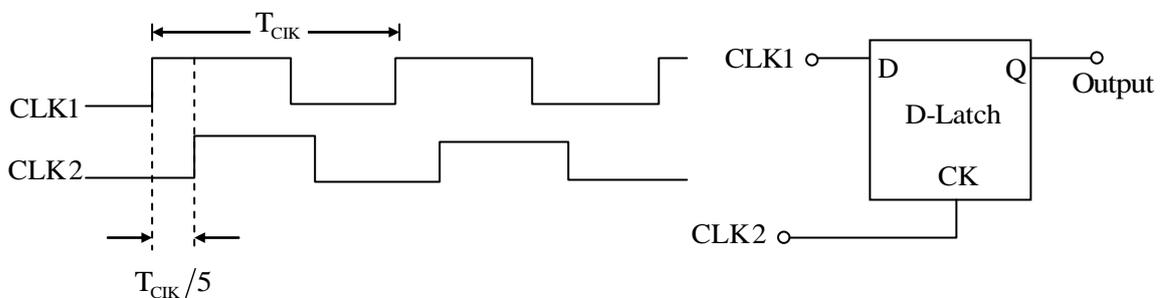
9. The Miller effect in the context of a Common Emitter amplifier explains \_\_\_\_\_

- (A) an increase in the low-frequency cutoff frequency  
 (B) an increase in the high-frequency cutoff frequency  
 (C) a decrease in the low-frequency cutoff frequency  
 (D) a decrease in the high-frequency cutoff frequency

**Key:** (D)

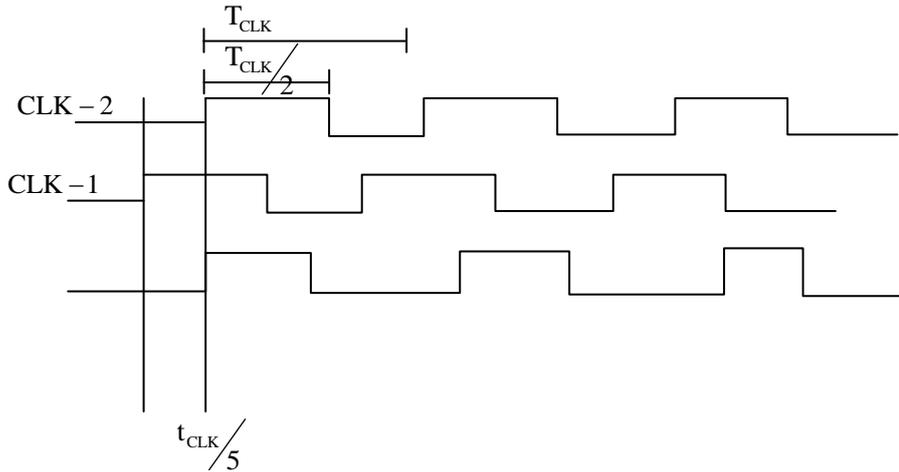
**Exp:** Miller effect increase input capacitance, so that there will be decrease in gain in the high frequency cutoff frequency.

10. Consider the D-Latch shown in the figure, which is transparent when its clock input CK is high and has zero propagation delay. In the figure, the clock signal CLK1 has a 50% duty cycle and CLK2 is a one-fifth period delayed version of CLK1. The duty cycle at the output latch in percentage is \_\_\_\_\_.



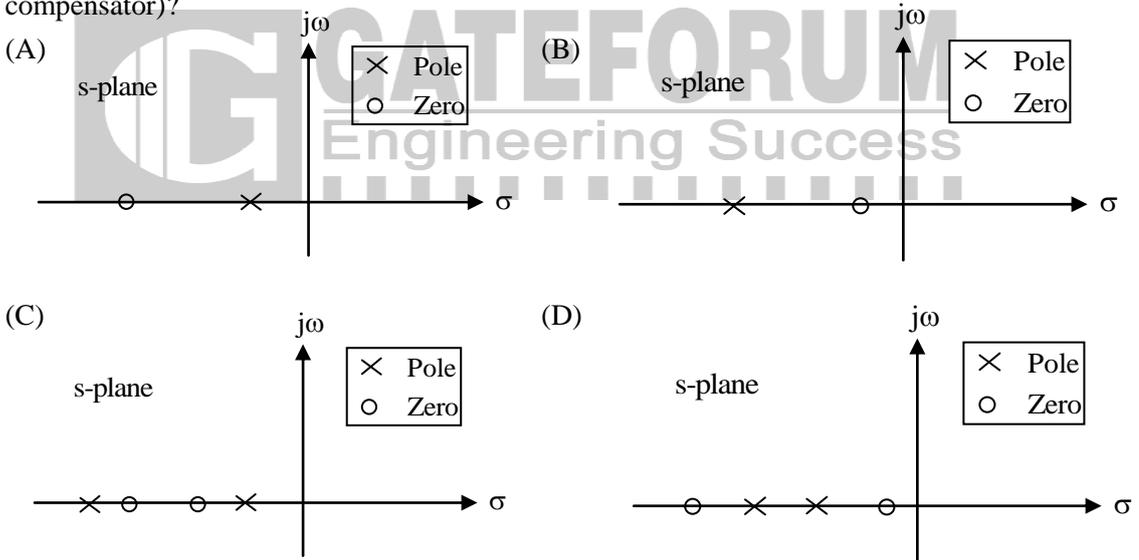
**Key:** (29.9 to 30.1)

**Exp:**



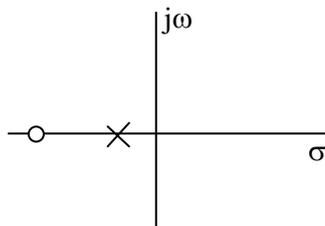
$$\Rightarrow \text{Duty of O/p} = \frac{\frac{T_{CLK}}{5} - T_{CLK}}{T_{CLK}} \times 100 = 30\%$$

11. Which of the following can be pole-zero configuration of a phase-lag controller (lag compensator)?



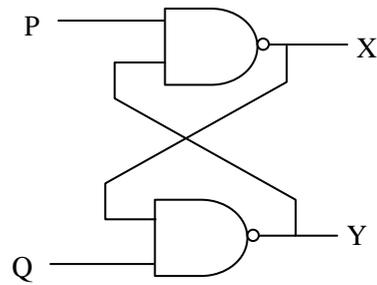
**Key:** (A)

**Exp:** In phase lag compensator pole is near to  $j\omega$ -axis,



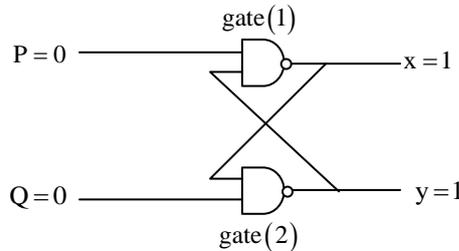
12. In the latch circuit shown, the NAND gates have non-zero, but unequal propagation delays. The present input condition is:  $P = Q = '0'$ . If the input condition is changed simultaneously to  $P = Q = '1'$ , the outputs X and Y are

- (A) X = '1', Y = '1'
- (B) either X = '1', Y = '0' or X = '0', Y = '1'
- (C) either X = '1', Y = '1' or X = '0', Y = '0'
- (D) X = '0', Y = '0'



**Key: (B)**

**Exp: Unequal propagation delay**



**Case I:**

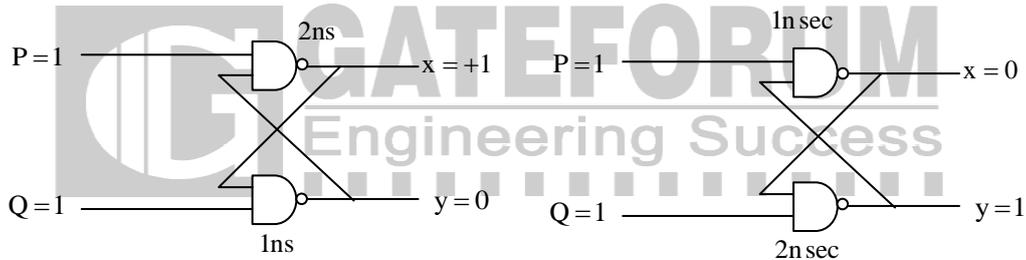
Gate 1 → 2ns

Gate 2 → 1ns

**Case II:**

Gate 1 → 1nsec

Gate 2 → 2nsec



∴ Either x = 1, y = 0 or x = 0, y = 1

13. Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) \_\_\_\_\_.

**Key: (0.027 to 0.028)**

**Exp:** Required probability =  $6 \left( \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \right) = \frac{1}{36} = 0.028$

14. A periodic signal x(t) has a trigonometric Fourier series expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t).$$

If  $x(t) = -x(-t) = -x(t - \pi/\omega_0)$ , we can conclude that

- (A)  $a_n$  are zero for all n and  $b_n$  are zero for n even
- (B)  $a_n$  are zero for all n and  $b_n$  are zero for n odd
- (C)  $a_n$  are zero for n even and  $b_n$  are zero for n odd
- (D)  $a_n$  are zero for n odd and  $b_n$  are zero for n even

**Key: (A)**

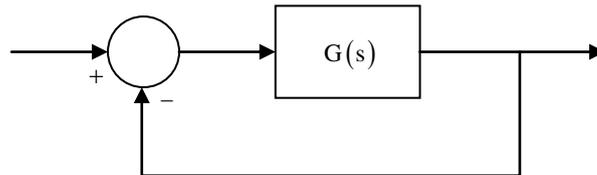
**Exp:** If  $x(t) = -x(-t)$  the given periodic signal is odd symmetric. For an odd symmetric signal  $a_n = 0$  for all n.

If  $x(t) = -x\left(t - \frac{\pi}{\omega_0}\right)$ ,  $\therefore \frac{\pi}{\omega_0} = \frac{T_0}{2}$  where  $T_0$  is fundamental period then the given condition satisfies half-wave symmetry.

For half-wave symmetrical signal all coefficients  $a_n$  and  $b_n$  are zero for even value of  $n$ .

15. The open loop transfer function  $G(s) = \frac{(s+1)}{s^p(s+2)(s+3)}$

Where  $p$  is an integer, is connected in unity feedback configuration as shown in figure.



Given that the steady state error is zero for unit step input and is 6 for unit ramp input, the value of the parameter  $p$  is \_\_\_\_\_.

**Key:** (0.99 to 1.01)

**Exp:**  $G(s) = \frac{s+1}{s^p(s+2)(s+3)}$

If  $p=1$ ,  $e_{ss}(\text{for ramp input}) = 6$

$$k_v = \frac{1}{6}$$

$p=1$   $e_{ss}(\text{for step input}) = 0$

$$k_p = \infty, e_{ss} = \frac{1}{1+k_p} = 0$$

16. An  $n^+ - n$  Silicon device is fabricated with uniform and non-degenerate donor doping concentrations of  $N_{D1} = 1 \times 10^{18} \text{ cm}^{-3}$  and  $N_{D2} = 1 \times 10^{15} \text{ cm}^{-3}$  corresponding to the  $n^+$  and  $n$  regions respectively. At the operational temperature  $T$ , assume complete impurity ionization,  $kT/q = 25 \text{ mV}$ , and intrinsic carrier concentration to be  $n_i = 1 \times 10^{10} \text{ cm}^{-3}$ . What is the magnitude of the built-in potential of this device?

- (A) 0.748V                      (B) 0.460V                      (C) 0.288V                      (D) 0.173V

**Key:** (D)

**Exp:**  $V_{bi} = V_T \ln\left(\frac{N_1}{N_2}\right)$   
 $= 0.25 \ln\left(\frac{10^{18}}{10^{15}}\right) = 0.173 \text{V}.$

17. For the operational amplifier circuit shown, the output saturation voltages are  $\pm 15 \text{V}$ . The upper and lower threshold voltages for the circuit are, respectively.





- (C) high input and output resistances
- (D) low input and output resistance

**Key:** (C)

**Exp:** A good trans conductance amplifier should have high input and output resistance.

22. Let  $(X_1, X_2)$  be independent random variables.  $X_1$  has mean 0 and variance 1, while  $X_2$  has mean 1 and variance 4. The mutual information  $I(X_1; X_2)$  between  $X_1$  and  $X_2$  in bits is \_\_\_\_\_.

**Key:** (0.0 to 0.0)

**Exp:** For two independent random variable

$$I(X; Y) = H(X) = H(X/Y)$$

$$H(X/Y) = H(X) \text{ for independent X and Y}$$

$$\Rightarrow I(X; Y) = 0$$

23. Consider the following statements for continuous-time linear time invariant (LTI) systems.

- I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.
- II. There is non causal and BIBO stable system with a pole in the right half of the complex plane.

Which one among the following is correct?

- (A) Both I and II are true
- (B) Both I and II are not true
- (C) Only I is true
- (D) Only II is true

**Key:** (D)

**Exp:** If a system is non-causal then a pole on right half of the s-plane can give BIBO stable system. But for a causal system to be BIBO all poles must lie on left half of the complex plane.

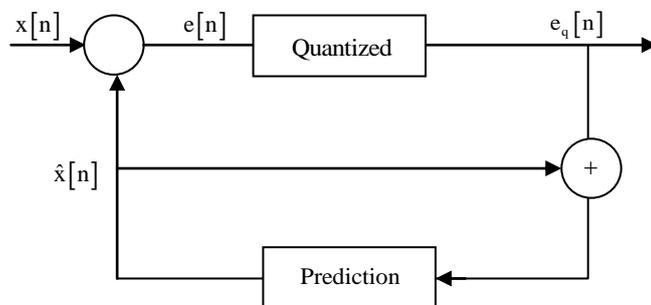
24. Which one of the following statements about differential pulse code modulation (DPCM) is true?

- (A) The sum of message signal sample with its prediction is quantized
- (B) The message signal sample is directly quantized, and its prediction is not used
- (C) The difference of message signal sample and a random signal is quantized
- (D) The difference of message signal sample with its predictions is quantized

**Key:** (D)

**Exp:** DPCM Block diagram

$e_q[n]$  is quantized  $e[n]$   
 $e[n]$  is difference of message signal sample with its prediction.



25. Consider a wireless communication link between a transmitter and a receiver located in free space, with finite and strictly positive capacity. If the effective areas of the transmitter and the

receiver antennas, and the distance between them are all doubled, and everything else remains unchanged, the maximum capacity of the wireless link

- (A) increases by a factor of 2                      (B) decrease by a factor 2  
(C) remains unchanged                              (D) decreases by a factor of  $\sqrt{2}$

**Key: (C)**

**Exp:**

$$C = B \log_2 \left[ 1 + \frac{S}{N_0 B} \right]$$

where  $S = \frac{P_t G_t A_{er}}{4\pi r^2}$

$$\begin{aligned} S^1 &= \frac{P_t A_{er} \cdot 4\pi}{4\pi r^2 \lambda^2} A_e t \\ &= \frac{P_t A_{er} A_e t}{\lambda^2 (r^2)} = P_t \frac{4 A_{er} \cdot A_e t}{4 \lambda^2 r^2} \\ &= \frac{P_t \cdot A_{er} \cdot A_e t}{A^2 r^2} = S \end{aligned}$$

Channel capacity remain same.

**Q. No. 26 to 55 Carry Two Marks Each**

26. Starting with  $x = 1$ , the solution of the equation  $x^3 + x = 1$ , after two iterations of Newton-Raphson's method (up to two decimal places) is \_\_\_\_\_.

**Key: (0.65 to 0.72)**

**Exp:** Let  $f(x) = x^3 + x - 1 \Rightarrow f'(x) = 3x^2 + 1$

Given  $x_0 = 1$

By Newton Raphson method,

$$1^{st} \text{ iteration, } x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

$$2^{nd} \text{ iteration, } x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.75 - \frac{f(0.75)}{f'(0.75)} = 0.75 - \frac{0.17}{2.69} = 0.69$$

27. In binary frequency shift keying (FSK), the given signal waveform are

$$u_0(t) = 5 \cos(20000\pi t); 0 \leq t \leq T, \text{ and}$$

$$u_1(t) = 5 \cos(22000\pi t); 0 \leq t \leq T,$$

Where T is the bit-duration interval and t is in seconds. Both  $u_0(t)$  and  $u_1(t)$  are zero outside the interval  $0 \leq t \leq T$ . With a matched filter (correlator) based receiver, the smallest positive value of T (in milliseconds) required to have  $u_0(t)$  and  $u_1(t)$  uncorrelated is

- (A) 0.25 ms                      (B) 0.5 ms                      (C) 0.75 ms                      (D) 1.0 ms

**Key: (B)**

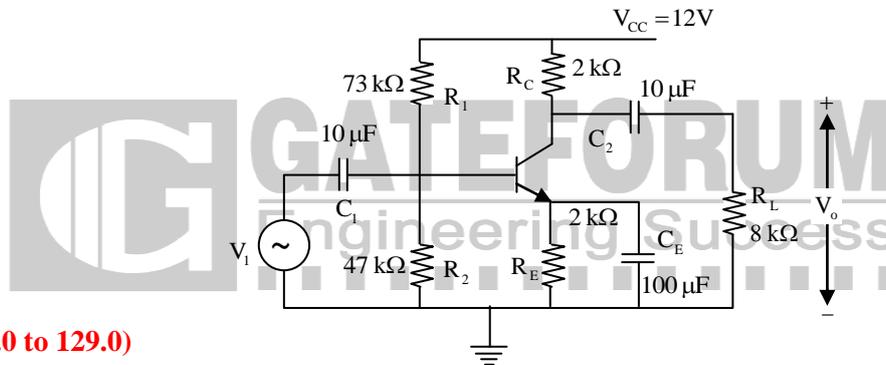
**Exp:**  $u_o(t) = 5 \cos(20000\pi t)$   
 $f_o = 10 \text{ kHz}$   
 $u_1(t) = 5 \cos(22000\pi t)$   
 $f_1 = 11 \text{ kHz}$

For  $u_o(t)$  and  $u_1(t)$  to be orthogonal, it is necessary that

$$f_1 - f_o = \frac{n}{2T}; \quad (11 - 10) \times 10^3 = \frac{1}{2T}$$

$$\Rightarrow T = \frac{1}{2 \times 10^3} = 0.5 \text{ msec}$$

28. For the DC analysis of the Common-Emitter amplifier shown, neglect the base current and assume that the emitter and collector current are equal. Given that  $V_T = 25\text{mV}$ ,  $V_{BE} = 0.7\text{V}$ , and the BJT output resistance  $r_o$  is practically infinite. Under these conditions, the midband voltage gain magnitude.  $A_v = |V_o/V_i| \text{ V/V}$ , is \_\_\_\_\_.



**Key:** (127.0 to 129.0)

**Exp:**  $A_v = \left| \frac{V_o}{V_i} \right| = \frac{R_c}{r_e}$

$$r_e = \frac{V_T}{I_E}$$

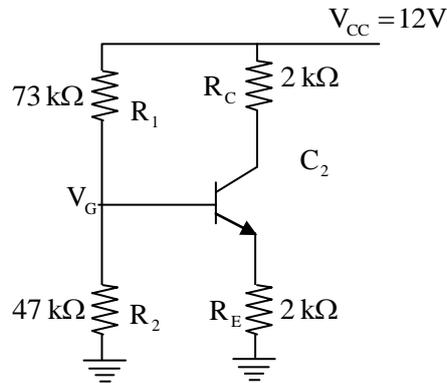
$$V_G = \frac{12 \times 47}{120} = 4.7 \text{ V}$$

$$V_G = V_{BE} + I_E R_E$$

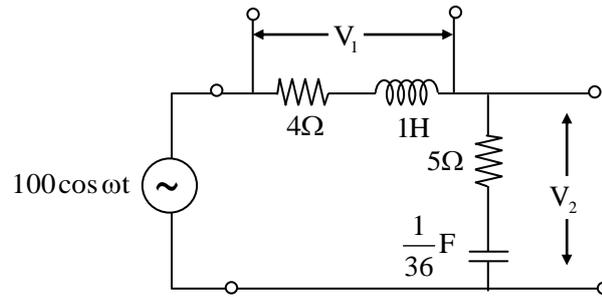
$$I_E = \frac{4.7 - 0.7}{2 \times 10^3} = 2 \text{ mA}$$

$$r_e = \frac{25}{2} = 12.5 \Omega$$

$$A_v = \frac{R_c \parallel R_L}{r_e} = \frac{2 \times 10^3 \parallel 8 \times 10^3}{12.5} = 128$$



29. The figure shows an RLC circuit excited by the sinusoidal voltage  $100\cos(3t)$  volts, where  $t$  is in seconds. The ratio  $\frac{\text{amplitude of } V_2}{\text{amplitude of } V_1}$  is \_\_\_\_\_.



**Key:** (2.55 to 2.65)

**Exp:**  $V_1 = \left( \frac{4 + j3}{4 + j3 + 5 - 12j} \right) \times 100 \angle 0 \Rightarrow V_1 = \left( \frac{4 + j3}{9 - 9j} \right) \times 100 \angle 0$

$$V_2 = \left( \frac{5 - 12j}{4 + j3 + 5 - 12j} \right) \times 100 \angle 0 \Rightarrow V_2 = \left( \frac{5 - 12j}{9 - 9j} \right) \times 100 \angle 0$$

$$\left| \frac{V_2}{V_1} \right| = \left| \frac{5 - 12j}{4 + j3} \right| = \frac{\sqrt{5^2 + 12^2}}{\sqrt{4^2 + 3^2}} = \frac{13}{5} = 2.6$$

30. Which one of the following is the general solution of the first order differential equation

$$\frac{dy}{dx} = (x + y - 1)^2, \text{ where } x, y \text{ are real?}$$

- (A)  $y = 1 + x + \tan^{-1}(x + c)$ , where  $c$  is a constant
- (B)  $y = 1 + x + \tan(x + c)$ , where  $c$  is a constant
- (C)  $y = 1 - x + \tan^{-1}(x + c)$ , where  $c$  is a constant
- (D)  $y = 1 - x + \tan(x + c)$ , where  $c$  is a constant

**Key:** (D)

**Exp:**  $\frac{dy}{dx} = (x + y - 1)^2 \quad \dots(1)$

Put  $x + y - 1 = t$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

From (1),  $\frac{dt}{dx} - 1 = t^2$

$$\Rightarrow \frac{dt}{dx} = 1 + t^2$$

$$\Rightarrow \int \frac{1}{1+t^2} dt = \int dx$$

$$\Rightarrow \tan^{-1}(t) = x + C$$

$$\Rightarrow \tan^{-1}(x + y - 1) = x + C$$

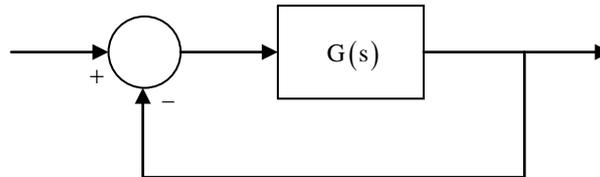
$$\Rightarrow x + y - 1 = \tan(x + C)$$

$$\Rightarrow y = 1 - x + \tan(x + C)$$

31. A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

is connected in unity feedback configuration as shown in the figure.



For the closed loop system shown, the root locus for  $0 < K < \infty$  intersects the imaginary axis for  $K = 1.5$ . The closed loop system is stable for

- (A)  $K > 1.5$       (B)  $1 < K < 1.5$       (C)  $0 < K < 1$       (D) no positive value of  $K$

**Key:** (A)

**Exp:** Given  $G(s) = \frac{k(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$

$$C.E = 1 + G(s) = s^2 - 3s + 2 + ks^2 + 2ks + 2k = 0$$

$$= s^2(1+k) + s(2k-s) + 2k + 2 = 0$$

If closed loop system to be stable all coefficients to positive

$$k > -1 \cap k > 1.5 \cap k > -1$$

So,  $k > 1.5$

32. Let  $I = \int_C (2z dx + 2y dy + 2x dz)$  where  $x, y, z$  are real, and let  $C$  be the straight line segment from point  $A : (0, 2, 1)$  to point  $B : (4, 1, -1)$ . The value of  $I$  is \_\_\_\_\_.

**Key:** (-11.1 to -10.9)

**Exp:** The straight line joining  $A(0, 2, 1)$  and  $B(4, 1, -1)$  is

$$\frac{x-0}{4-0} = \frac{y-2}{1-2} = \frac{z-1}{-1-1}$$

$$\Rightarrow \frac{x}{4} = \frac{y-2}{-1} = \frac{z-1}{-2} = t \text{ (say)}$$

$$\Rightarrow x = 4t, y = 2 - t, z = 1 - 2t$$

$$\Rightarrow dx = 4dt, dy = -dt, dz = -2dt$$

$$\text{For } x = 0 \Rightarrow t = 0$$

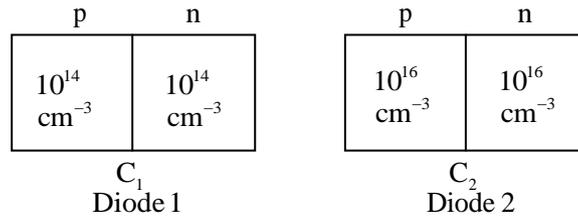
$$\text{For } x = 4 \Rightarrow t = 1$$

$$I = \int_C (2zdx + 2ydy + 2xdz)$$

$$= \int_{t=0}^1 2(1-2t)4dt + 2(2-t)(-dt) + 2(4t)(-2dt)$$

$$\therefore = \int_{t=0}^1 (-30t + 4) dt = \left. \frac{-30t^2}{2} + 4t \right|_0^1 = -11$$

33. As shown, two Silicon (Si) abrupt p-n junction diodes are fabricated with uniform donor doping concentration of  $N_{D1} = 10^{14} \text{ cm}^{-3}$  and  $N_{D2} = 10^{16} \text{ cm}^{-3}$  in the n-regions of the diodes, and uniform acceptor doping concentration of  $N_{A1} = 10^{14} \text{ cm}^{-3}$  and  $N_{A2} = 10^{16} \text{ cm}^{-3}$  in the p-regions of the diodes, respectively. Assuming that the reverse bias voltage is  $\gg$  built-in potentials of the diodes, the ratio  $C_2/C_1$  of their reverse bias capacitances for the same applied reverse bias, is \_\_\_\_\_.



**Key: (10.0 to 10.0)**

**Exp:**  $C = \frac{\epsilon A}{W}$

$C \propto \frac{1}{W}$  and  $W \propto \frac{1}{\sqrt{\text{doping}}}$

$C \propto \sqrt{\text{doping}}$

$$\frac{C_2}{C_1} = \sqrt{\frac{(\text{doping})_2}{(\text{doping})_1}} = \sqrt{\frac{10^{16}}{10^{14}}} = \sqrt{100} = 10$$

34. An optical fiber is kept along the  $\hat{z}$  direction. The refractive indices for the electric fields along  $\hat{x}$  and  $\hat{y}$  directions in the fiber are  $n_x = 1.5000$  and  $n_y = 1.5001$ , respectively ( $n_x \neq n_y$  due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is  $1.5 \mu\text{m}$ . If the light wave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized, in centimeter, is \_\_\_\_\_.

**Key: (0.36 to 0.38)**

**Exp:** For circular polarization the phase difference between  $E_x$  &  $E_y$  is  $\pi/2$

$\Rightarrow$  The phase difference for linear polarization should be  $\pi$

So the wave must travel a minimum distance such that the extra phase difference of  $\pi/2$  must occur.

$$\beta_y \ell_{\min} - \beta_x \ell_{\min} = \pi/2$$

$$\Rightarrow \ell_{\min} \frac{\omega}{c} [n_y - n_x] = \pi/2 \Rightarrow \frac{2\pi \ell_{\min}}{\lambda_o} [n_y - n_x] = \pi/2$$

$$\Rightarrow \ell_{\min} = \frac{\lambda_o}{4[n_y - n_x]} = \frac{1.5 \times 10^{-6}}{4[0.0001]} = \frac{1.5}{4} \times 10^{-2} = 0.375 \times 10^{-2} \text{ m} = 0.375 \text{ cm}$$

35. Two discrete-time signals  $x[n]$  and  $h[n]$  are both non-zero for  $n = 0, 1, 2$  and are zero otherwise. It is given that

$$x[0]=1, x[1]=2, x[2]=1, h[0]=1.$$

Let  $y[n]$  be the linear convolution of  $x[n]$  and  $h[n]$ . Given that  $y[1] = 3$  and  $y[2] = 4$ , the value of the expression  $(10y[3] + y[4])$  is \_\_\_\_\_.

**Key:** (31.00 to 31.00)

**Exp:** Given

$$x[n] = \{1, 2, 1\}; \because h[0] = 1$$

$$h[n] = \{1, a, b\}$$

	1	2	1
1	1	2	1
a	a	2a	a
b	b	2b	b

$$y[n] = \{1, 2 + a, 2a + b + 1, 2b + a, b\}$$

It is given that  $y[1] = 3$

$$\therefore 2 + a = 3 \Rightarrow a = 1$$

Similarly  $2a + b + 1 = 4 \Rightarrow b = 3 - 2(1) = 1$

$$b = 1$$

$$\therefore y[3] = 2(1) + 1 = 3$$

$$y[4] = b = 1$$

$$\therefore 10y[3] + y[4] = 30 + 1 = 31$$

36. Which one of the following options correctly describes the locations of the roots of the equation  $s^4 + s^2 + 1 = 0$  on the complex plane?

- (A) Four left half plane (LHP) roots
- (B) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis
- (C) Two RHP roots and two LHP roots
- (D) All four roots are on the imaginary axis

**Key:** (C)

**Exp:**  $F(s) = s^4 + s^2 + 1 = 0$

Let take  $s^2 = t$

$$t^2 + t + 1 = 0$$

$$t = \frac{-1 \pm i\sqrt{3}}{2}$$

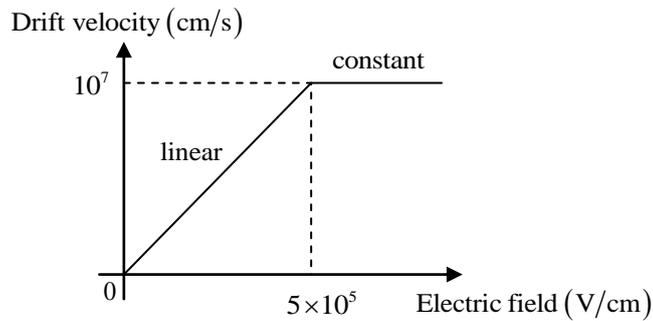
$$\text{Where } t = s^2 = \frac{-1 \pm j\sqrt{3}}{2}$$

$$s^2 = \frac{-1 + j\sqrt{3}}{2} = e^{j\frac{2\pi}{3}} \Rightarrow s = \frac{-1 - j\sqrt{3}}{2} = e^{-j\frac{2\pi}{3}}$$

$$s = \pm e^{\frac{j2\pi}{6}} \text{ and } s = \pm e^{-\frac{j2\pi}{6}}$$

Hence two roots contain RHS and two roots contain LHS plane.

37. The dependence of drift velocity of electrons on electric field in a semiconductor is shown below. The semiconductor has a uniform electron concentration of  $n = 1 \times 10^{16} \text{ cm}^{-3}$  and electronic charge  $q = 1.6 \times 10^{-19} \text{ C}$ . If a bias of 5V is applied across a  $1 \mu\text{m}$  region of this semiconductor, the resulting current density in this region, in  $\text{kA/cm}^2$ , is \_\_\_\_\_.



**Key: (1.5 to 1.7)**

**Exp:**  $v_d = \mu_n \epsilon$

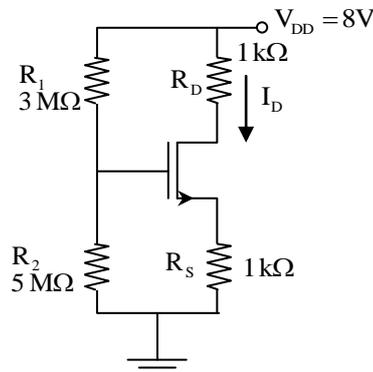
$$\mu_n = \frac{v_d}{\epsilon} = \frac{10^7}{5 \times 10^5} = 20 \frac{\text{cm}^2}{\text{V-sec}}$$

$$E = \frac{V}{d} = \frac{5}{1 \times 10^{-4}} \text{ V/cm}$$

$$J_{\text{drift}} = nqv_d = nq\mu_n \epsilon$$

$$J_{\text{drift}} = nqv_a = nq\mu_n \epsilon = 10^{16} \times 1.6 \times 10^{-19} \times 20 \times 5 \times 10^4 = 1.6 \text{ KA/cm}^2$$

38. For the circuit shown, assume that the NMOS transistor is in saturation. Its threshold voltage  $V_{tn} = 1\text{V}$  and its transconductance parameter  $\mu_n C_{ox} \left(\frac{W}{L}\right) = 1 \text{ mA/V}^2$ . Neglect channel length modulation and body bias effects. Under these conditions, the drain current  $I_D$  in mA is \_\_\_\_\_.



**Key: (1.9 to 2.1)**

**Exp:**  $V_G = \frac{8 \times 5}{8} = 5\text{V}$

$$V_{GS} = V_G - I_D R_S = 5 - 10^3 I_D$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_T)^2$$

$$I_D = \frac{1}{2} \times 1 \times 10^{-3} (V_{GS} - V_T)^2$$

$$I_D = \frac{1}{2} \times 10^{-3} [5 - 10^3 I_D - 1]^2 = \frac{10^{-3}}{2} [4 - 10^3 I_D]^2$$

$$I_D = \frac{10^{-3}}{2} [16 + 10^6 I_D^2 - 8 \times 10^3]$$

$$0.5 \times 10^3 I_D^2 - 5 I_D + 8 \times 10^{-3} = 0$$

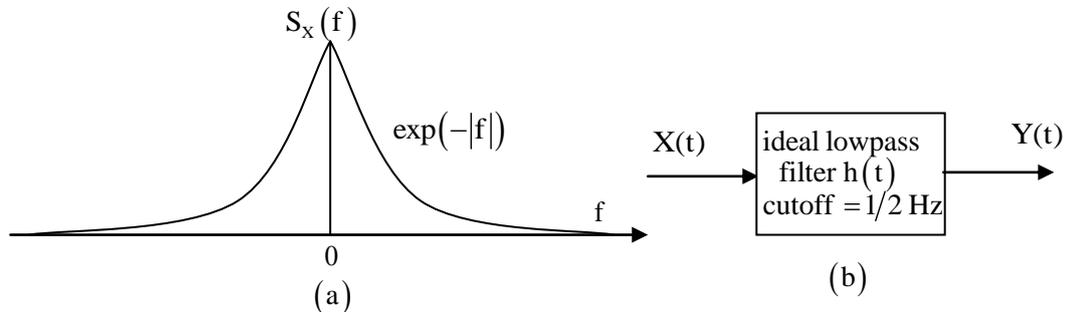
$$I_D = 8\text{mA}, 2\text{mA}$$

$I_D$  must be least value  
So  $I_D = 2\text{mA}$

39. Let  $X(t)$  be a wide sense stationary random process with the power spectral density  $S_X(f)$  as shown in Figure (a), where  $f$  is in Hertz (Hz). The random process  $X(t)$  is input to an ideal low pass filter with frequency response

$$H(f) = \begin{cases} 1, & |f| \leq \frac{1}{2} \text{ Hz} \\ 0, & |f| > \frac{1}{2} \text{ Hz} \end{cases}$$

As shown in Figure (b). The output of the lowpass filter is  $Y(t)$ .



Let  $E$  be the expectation operator and consider the following statements.

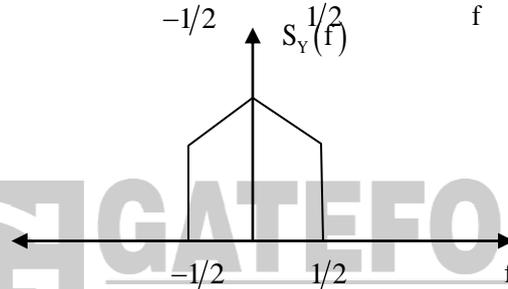
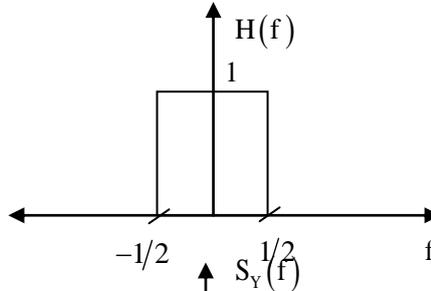
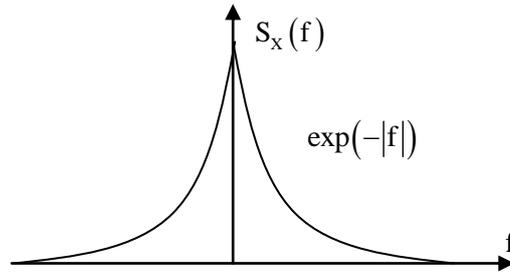
- I.  $E(X(t)) = E(Y(t))$
- II.  $E(X^2(t)) = E(Y^2(t))$
- III.  $E(Y^2(t)) = 2$

Select the correct option:

- (A) only I is true
- (B) only II and III are true
- (C) only I and II are true
- (D) only I and III are true

**Key: (A)**

**Exp:**



Since DC components in same in  $S_x(f)$  and  $S_y(f)$

$$\Rightarrow E[x(t)] = E[y(t)]$$

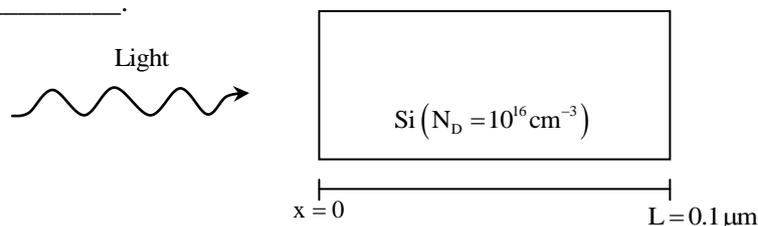
$$E[x^2(t)] = \text{Area under } S_x(f) = \int_{-\infty}^{\infty} e^{-|f|} df = 2 \int_0^{\infty} e^{-f} df = 2$$

$$E[y^2(t)] = \text{Area under } S_y(f) = 2 \int_0^{1/2} e^{-f} df = 2 \left[ \frac{e^{-f}}{-1} \right]_0^{1/2} = 2[1 - e^{-1/2}]$$

$$\Rightarrow E[x^2(t)] \neq E[y^2(t)]$$

$$E[y^2(t)] \neq 2$$

40. As shown a uniformly doped Silicon (Si) bar of length  $L = 0.1 \mu\text{m}$  with a donor concentration  $N_D = 10^{16} \text{cm}^{-3}$  is illuminated at  $x = 0$  such that electron and hole pairs are generated at the rate of  $G_L = G_{L0} \left(1 - \frac{x}{L}\right), 0 \leq x \leq L$ , where  $G_{L0} = 10^{17} \text{cm}^{-3}\text{s}^{-1}$ . Hole lifetime is  $10^{-4} \text{s}$ , electronic charge  $q = 1.6 \times 10^{-19} \text{C}$ , hole diffusion coefficient  $D_p = 100 \text{cm}^2/\text{s}$  and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at  $x = L$ , the magnitude of the diffusion current density at  $x = L/2$ , in  $\text{A}/\text{cm}^2$ , is \_\_\_\_\_.



**Key: (15.9 to 16.1)**

**Exp:**  $\Delta P = \Delta n = G_{Lo} \left( 1 - \frac{L}{L} \right) \tau_p = G_{Lo} \times \frac{1}{2} \times \tau_p = 10^{17} \times \frac{1}{2} \times 10^{-4} = \frac{1}{2} \times 10^{13} / \text{cm}^3$

$$\left| J_{P_i \text{ diff}} \right| = qD_p \frac{dp}{dx} = 1.6 \times 10^{-19} \times 100 \times \frac{\frac{1}{2} \times 10^{13}}{\frac{0.1 \times 10^{-4}}{2}} = 16 \text{A} / \text{cm}^2$$

41. The Nyquist plot of the transfer function

$$G(s) = \frac{K}{(s^2 + 2s + 2)(s + 2)}$$

does not encircle the point  $(1 + j0)$  for  $K = 10$  but does encircle the point  $(-1 + j0)$  for  $K = 100$ . Then the closed loop system (having unity gain feedback) is

- (A) stable for  $K = 10$  and stable for  $K = 100$
- (B) stable for  $K = 10$  and unstable for  $K = 100$
- (C) unstable for  $K = 10$  and stable for  $K = 100$
- (D) unstable for  $K = 10$  and unstable for  $K = 100$

**Key: (B)**

**Exp:**  $G(s) = \frac{k}{(s^2 + 2s + 2)(s + 2)}$

$$C.E = s^3 + 4s^2 + 76s + 4 + k = 0$$

If system to stable

$$24 > k + 4 \cap k + 4 > 0$$

$$k > -4 \cap k < 20$$

(i) Stable condition  $-4 < k < 20$

Means If  $k = 10$  system stable

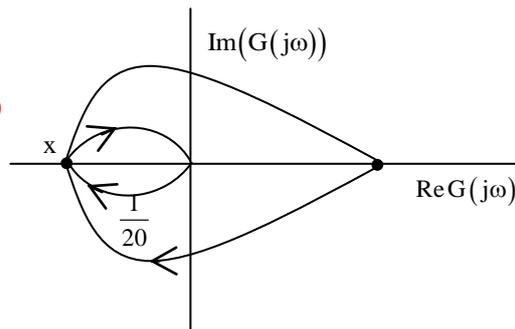
$k = 100$  system unstable

Or  $G(j\omega) = \frac{k}{(2 - \omega^2 + 2j\omega)(2 + j\omega)}$

$$\angle G(j\omega) = \angle - \tan^{-1} \left( \frac{\omega}{2} \right) - \tan^{-1} \left( \frac{2\omega}{2 - \omega^2} \right)$$

If  $\omega \rightarrow 0$   $G(j\omega) = \frac{k}{4} < 0$

$\omega \rightarrow \infty$   $G(j\omega) = 0 \angle -270$



So If  $k = 10$  touching point = 0.5

If  $k = 100$  touching point = 5

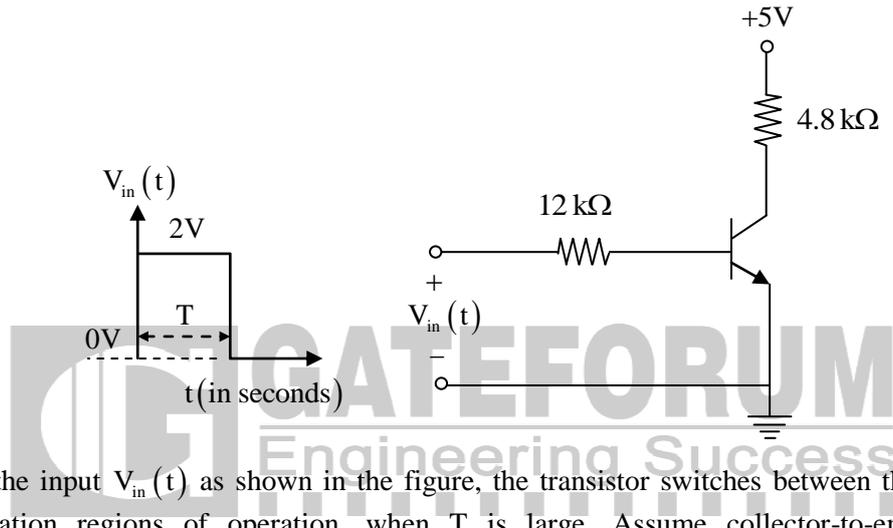
$N = P - Z$ , Here  $P = 0$

$N = -Z$

If closed loop system to be stable, then  $Z = 0, \Rightarrow N = 0$ ,

So,  $k = 10$  is stable system

42. In the figure shown, the npn transistor acts as a switch



For the input  $V_{in}(t)$  as shown in the figure, the transistor switches between the cut-off and saturation regions of operation, when  $T$  is large. Assume collector-to-emitter voltage saturation  $V_{CE(sat)} = 0.2V$  and base-to-emitter voltage  $V_{BE} = 0.7V$ . The minimum value of the common-base current gain ( $\alpha$ ) of the transistor for the switching should be \_\_\_\_\_.

**Key: (0.89 to 0.91)**

**Key:**  $I_B = \frac{(2 - 0.7)}{12 \times 10^3} = 0.108 \text{mA}$

$$I_C = \frac{5 - 0.2}{4.8 \times 10^3} = 1 \text{mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{1}{0.108} = 9.259$$

$$\alpha = \frac{\beta}{1 + \beta} = \frac{9.259}{1 + 9.259} = 0.903$$

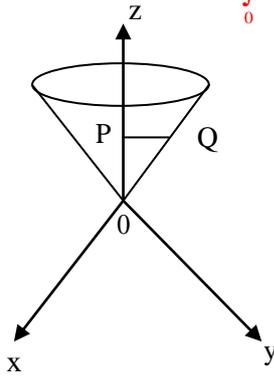
43. A three dimensional region  $R$  of finite volume is described by  $x^2 + y^2 \leq z^3; 0 \leq z \leq 1$ , Where  $x, y, z$  are real. The volume of  $R$  (up to two decimal places) is \_\_\_\_\_.

**Key: (0.70 to 0.85)**

**Exp:**  $PQ = \sqrt{x^2 + y^2}$  is the radius of variable circle at some  $Z$ .

$$\Rightarrow PQ^2 = x^2 + y^2 = z^3 \quad (\text{Given})$$

$$\therefore \text{Volume of region revolved around } z\text{-axis} = \int_0^1 \pi(PQ)^2 dz = \int_0^1 \pi z^3 dz = \pi \frac{z^4}{4} \Big|_0^1 = \frac{\pi}{4} = 0.79$$



44. The expression for an electric field in free space is  $E = E_0 = (\hat{x} + \hat{y} + j2\hat{z})e^{-j(\omega t - kx + ky)}$ , where  $x, y, z$  represent the spatial coordinates,  $t$  represents time, and  $\omega, k$  are constants. This electric field
- (A) does not represent a plane wave
  - (B) represents a circular polarized plane wave propagating normal to the  $z$ -axis
  - (C) represents an elliptically polarized plane wave propagating along  $x$ - $y$  plane.
  - (D) represents a linearly polarized plane wave

**Key:** (C)

**Exp:** Given the direction of propagation is  $\hat{x} - \hat{y}$

The orientation of  $\vec{E}$  field is  $\hat{x} + \hat{y} + j2\hat{z}$

The dot product between above two is  $= 1 - 1 + 0 = 0$

$\Rightarrow$  It is a plane wave

We observed that

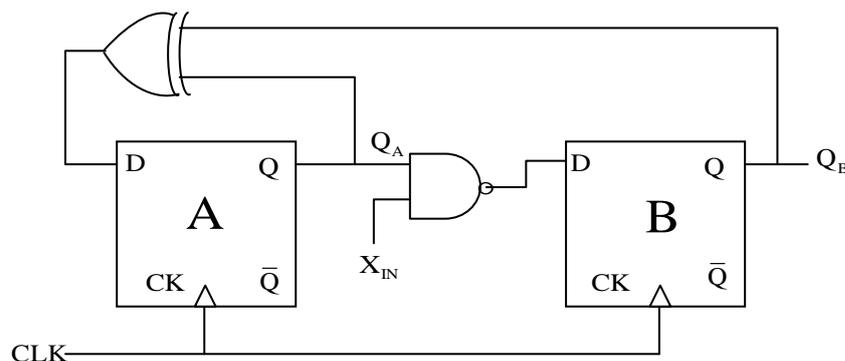
$\vec{P} = \hat{x} - \hat{y}, \hat{x} + \hat{y}$  and  $j2\hat{z}$  are normal to each other.

So electric field can be resolved into two normal component along  $\hat{x} + \hat{y}$  and  $j2\hat{z}$

The magnitude are  $\sqrt{2}$  and  $2$  and  $\theta = \frac{\pi}{2}$

So elliptical polarization.

45. A finite state machine (FSM) is implemented using the D flip-flops A and B, and logic gates, as shown in the figure below. The four possible states of the FSM are  $Q_A Q_B = 00, 01, 10$  and  $11$ .



Assume that  $X_{IN}$  is held at a constant logic level throughout the operation of the FSM. When the FSM is initialized to the state  $Q_A Q_B = 00$  and clocked, after a few clock cycles, it starts cycling through

- (A) all of the four possible states if  $X_{IN} = 1$
- (B) three of the four possible states if  $X_{IN} = 0$
- (C) only two of the four possible states if  $X_{IN} = 1$
- (D) only two of the four possible states if  $X_{IN} = 0$

**Key: (D)**

**Exp:** In given diagram

Prsent State	$D_A$	$D_B$	$X_{in}$	$X_{in}$	$X_{in}=0$ Next State		$X_{in}=1$ Next State	
					$\theta_A^+$	$\theta_B^+$	$\theta_A^+$	$\theta_B^+$
00	0	1	0	1	0	1	0	1
01	1	1	0	1	1	1	1	1
11	0	1	0	1	0	1	0	0
01	1	1	0	1	1	1	0	1

When  $X_{in} = 0$  2State

When  $X_{in}=1$  3 State

46. Let  $x(t)$  be a continuous time periodic signal with fundamental period  $T = 1$  seconds. Let  $\{a_k\}$  be the complex Fourier series coefficients of  $x(t)$ , where  $k$  is integer valued. Consider the following statements about  $x(3t)$ :

- I. The complex Fourier series coefficients of  $x(3t)$  are  $\{a_k\}$  where  $k$  is integer valued
- II. The complex Fourier series coefficients of  $x(3t)$  are  $\{3a_k\}$  where  $k$  is integer valued
- III. The fundamental angular frequency of  $x(3t)$  is  $6\pi$  rad/s

For the three statements above, which one of the following is correct?

- (A) only II and III are true (B) only I and III are true
- (C) only III is true (D) only I is true

**Key: (B)**

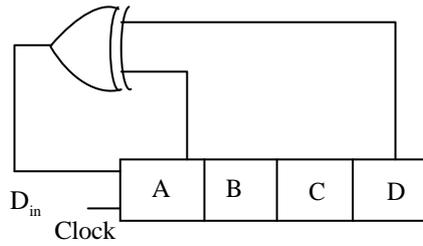
**Exp:** Fourier series coefficient  $a_k$  is unaffected by scaling operating. Thus (I) is true and (II) is false.

$T = 1\text{sec}$  for  $x(t)$  and if it compressed by '3' then the resultant period  $T = \frac{1}{3}$

$\therefore$  Fundamental frequency  $= \frac{2\pi}{T_1} = 6\pi$  rad/sec.

Thus (III) is correct.

47. A 4-bit shift register circuit configured for right-shift operation, i.e,  $D_{in} \rightarrow A, A \rightarrow B, B \rightarrow C, C \rightarrow D$ , is shown. If the present state of the shift register is  $ABCD = 1101$ , the number of clock cycles required to reach the state  $ABCD = 1111$  is \_\_\_\_\_.



**Key:** (10.0 to 10.0)

**Exp:**

CLK	A	B	C	D	
	$D_{in} = A \oplus B$	$A \rightarrow B$	$B \rightarrow C$	$C \rightarrow D$	
0	1	1	0	1	→ initial state
1	0	1	1	0	
2	0	0	1	1	
3	1	0	0	1	
4	0	1	0	0	
5	0	0	1	0	
6	0	0	0	1	
7	1	0	0	0	
8	1	1	0	0	
9	1	1	1	0	
10	1	1	1	1	→ Final state

10 clock pulse required.

48. Let  $f(x) = e^{x+x^2}$  for real  $x$ . From among the following, choose the Taylor series approximation of  $f(x)$  around  $x = 0$ , which included all powers of  $x$  less than or equal to 3.

- (A)  $1+x+x^2+x^3$  (B)  $1+x+\frac{3}{2}x^2+x^3$   
 (C)  $1+x+\frac{3}{2}x^2+\frac{7}{6}x^3$  (D)  $1+x+3x^2+7x^3$

**Key:** (C)

**Exp:** We have Taylor series of  $f(x)$  around  $x = 0$  is  $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)$

(upto powers of 'x' less than or equal to '3')

Given

$$f(x) = e^{x+x^2} \Rightarrow f(0) = 1$$

$$f'(x) = e^{x+x^2}(1+2x) \Rightarrow f'(0) = 1$$

$$f''(x) = e^{x+x^2}(1+2x)^2 + 2e^{x+x^2} \Rightarrow f''(0) = 3$$

$$f'''(x) = e^{x+x^2}(1+2x)^3 + e^{x+x^2}4(1+2x) + 2(1+2x)e^{x+x^2} \Rightarrow f'''(0) = 7$$

$$\therefore f(x) = e^{x+x^2} = 1 + x.1 + \frac{x^2}{2!}(3) + \frac{x^3}{3!}(7) = 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$$

49. The following FIVE instructions were executed on an 8085 microprocessor.

MVI A, 33H  
MVI B, 78H  
ADD B  
CMA  
ANI 32H

The Accumulator value immediately after the execution of the fifth instruction is

- (A) 00H                      (B) 10H                      (C) 11H                      (D) 32H

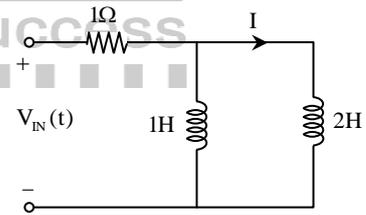
**Key: (B)**

**Exp:** MVI A, 33H    A ← 33H  
MVI B, 78H    B ← 78H  
ADD B        B ← ABH  
CMA         A ← 54H  
ANI 32H     A ← 10H

A → 0011 0011	A → 1010 1011	0101 0100
B → 0111 1000	B → 0101 0100	0011 0010
1010 1011		0001 0000

50. In the circuit shown, the voltage  $V_{IN}(t)$  is described by:

$$V_{IN} = \begin{cases} 0, & \text{for } t < 0 \\ 15 \text{ volts} & \text{for } t \geq 0 \end{cases}$$

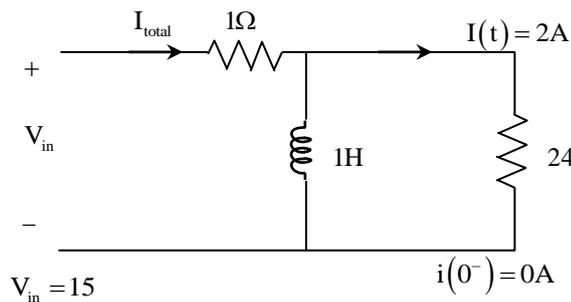


Where  $t$  is in seconds. The time (in seconds) at which the current  $I$  in the circuit will reach the value 2 Amperes is \_\_\_\_\_.

**Key: (0.30 to 0.40)**

**Exp:** Under dc condition inductor acts as short all

$$\begin{aligned} \therefore I_{total} &= \frac{15}{1} = 15A \\ i(t) &= (i(0^-) - i(\infty))e^{-t/\tau} + i(\infty) \\ i(0^-) &= i(0^+) = 0A \\ i(t) &= (0 - 15)e^{-3/2t} + 15 \\ i_{total}(t) &= 15(1 - e^{-3/2t})A \\ i(t)_{total} &= \frac{I_{total}}{3} = \frac{15}{3}(1 - e^{-3/2t}) \\ 2 &= 5(1 - e^{-3/2t}) \Rightarrow t = 0.34 \text{ sec} \end{aligned}$$



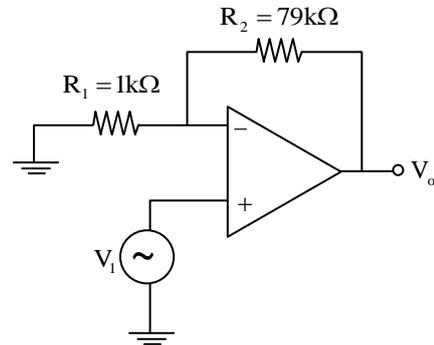
51. A half wavelength dipole is kept in the  $x$ - $y$  plane and oriented along  $45^\circ$  from the  $x$ -axis. Determine the direction of null in the radiation pattern for  $0 \leq \phi \leq \pi$ . Here the angle  $\theta$  ( $0 \leq \theta < \pi$ ) is measured from the  $z$ -axis, and the angle  $\phi$  ( $0 \leq \phi \leq 2\pi$ ) is measured from the  $x$ -axis in the  $x$ - $y$  plane.

- (A)  $\theta = 90^\circ, \phi = 45^\circ$  (B)  $\theta = 45^\circ, \phi = 90^\circ$   
 (C)  $\theta = 90^\circ, \phi = 135^\circ$  (D)  $\theta = 45^\circ, \phi = 135^\circ$

**Key:** (A)

**Exp:** The null occurs along axis of the antenna which is  $\theta = 90^\circ$  and  $\phi = 45^\circ$

52. The amplifier circuit shown in the figure is implemented using a compensated operational amplifier (op-amp), and has an open-loop voltage gain,  $A_o = 10^5$  V/V and an open-loop cut-off frequency  $f_c = 8$ Hz. The voltage gain of the amplifier at 15 kHz, in V/V is \_\_\_\_\_.



**Key:** (43.3 to 45.3)

**Exp:** Given Amplifier is using -ve feed back

$$A_f = \frac{A_o}{1 + A_o\beta}$$

$$\beta = \frac{1}{80}; A_o = 10^5$$

$$A_f = \frac{10^5}{1 + 10^5 / 80} = 79.93$$

$$f_{cut} = 8\text{Hz} \times (1 + A_o\beta) = 10008\text{Hz}$$

$$A_f(\omega) = \frac{A_f}{\sqrt{1 + (f / f_{cut})^2}}$$

$$= \frac{79.93}{\sqrt{1 + \left(\frac{15 \times 10^3}{10008}\right)^2}}$$

$$= 44.3$$

53. Let  $h[n]$  be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$h[0] = \frac{1}{3}; h[1] = \frac{1}{3}; h[2] = \frac{1}{3}; \text{ and } h[n] = 0 \text{ for } n < 0 \text{ and } n > 2.$$

Let  $H(\omega)$  be the discrete-time Fourier system transform (DTFT) of  $h[n]$ , where  $\omega$  is the normalized angular frequency in radians. Given that  $H(\omega_0) = 0$  and  $0 < \omega_0 < \pi$ , the value of  $\omega_0$  (in radians) is equal to \_\_\_\_\_.

**Key:** (2.05 to 2.15)

**Exp:** It is given that,

$$h[0] = \frac{1}{3}; h[1] = \frac{1}{3}; h[2] = \frac{1}{3} \&$$

$$h[n] = 0 \text{ for } n < 0 \text{ and } n > 2.$$

$$\therefore h[n] = h[0]\delta[n] + h[1]\delta[n-1] + h[2]\delta[n-2]$$

$$= \frac{1}{3}[\delta[n] + \delta[n-1] + \delta[n-2]]$$

Apply DTFT on both sides,

$$\therefore H(\omega) = \frac{1}{3}[1 + e^{-j\omega} + e^{-2j\omega}]$$

Given that  $H(\omega_0) = 0$  &  $0 < \omega_0 < \pi$

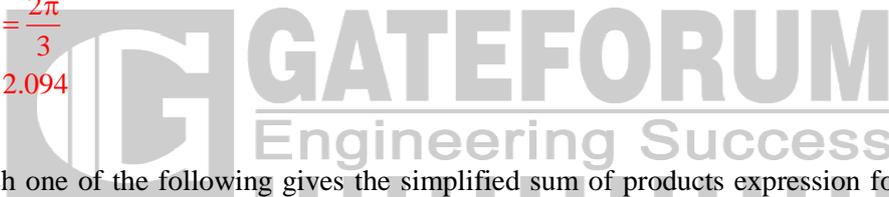
$$\Rightarrow H(\omega_0) = \frac{1}{3}\left[1 + e^{-\frac{3j\omega_0}{2}}\left(e^{\frac{-j\omega_0}{2}} + e^{\frac{j\omega_0}{2}}\right)\right] = 0$$

$$\therefore 1 + 2e^{-\frac{3j\omega_0}{2}} \cos\frac{\omega_0}{2} = 0$$

$$\text{consider } |H(\omega_0)| \Rightarrow \cos\frac{\omega_0}{2} = \frac{1}{2}$$

$$\therefore \omega_0 = \frac{2\pi}{3}$$

$$\omega_0 = 2.094$$



54. Which one of the following gives the simplified sum of products expression for the Boolean function  $F = m_0 + m_2 + m_3 + m_5$ , where  $m_0, m_2, m_3$  and  $m_5$  are minterms corresponding to the inputs A, B and C with A as the MSB and C as the LSB?

(A)  $\bar{A}B + \bar{A}\bar{B}\bar{C} + A\bar{B}C$

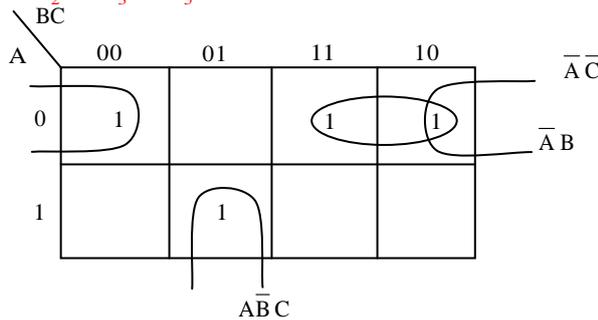
(B)  $\bar{A}\bar{C} + \bar{A}B + A\bar{B}C$

(C)  $\bar{A}\bar{C} + A\bar{B} + A\bar{B}C$

(D)  $\bar{A}BC + \bar{A}\bar{C} + A\bar{B}C$

**Key: (B)**

**Exp:**  $F = M_0 + M_2 + M_3 + M_5 \rightarrow \text{minterm}$



55. A continuous time signal  $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$ , where  $t$  is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response

$$h(t) = \begin{cases} \frac{2\sin(300\pi t)}{\pi t}, & t \neq 0 \\ 600, & t = 0 \end{cases}$$

Let  $y(t)$  be the output of this filter. The maximum value of  $|y(t)|$  is \_\_\_\_\_.

**Key: (7.90 to 8.10)**

**Exp:** Given

$$h(t) = \begin{cases} \frac{2 \sin 300\pi t}{\pi t}, & t \neq 0 \\ 600, & t = 0 \end{cases}$$

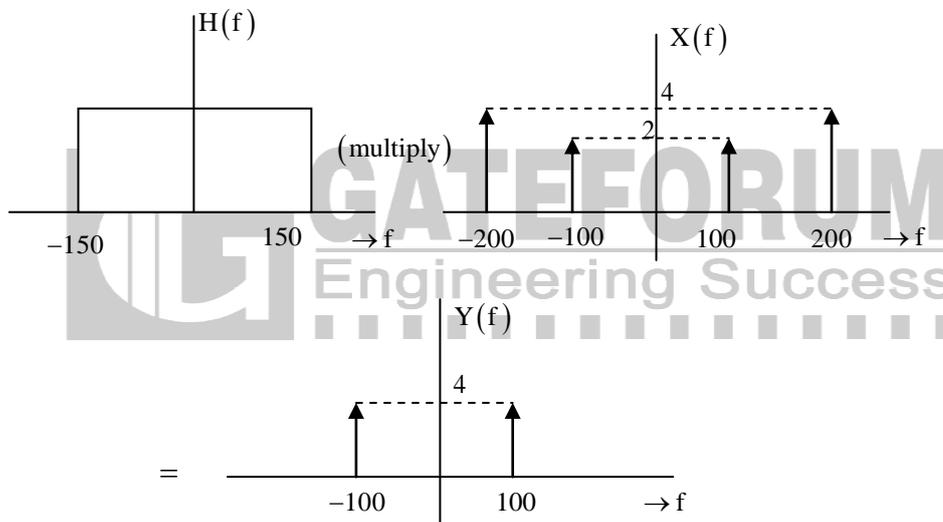
Thus  $h(t) = 600 \text{sinc}(300t)$

$$\therefore H(f) = 2 \text{rect}\left(\frac{f}{300}\right).$$

Given  $x(t) = 4 \cos 200\pi t + 8 \cos 400\pi t$

In  $f$ -domain,

$$X(f) = 2[\delta(f - 100) + \delta(f + 100)] + 4[\delta(f - 200) + \delta(f + 200)]$$



### General Aptitude

#### Q. No. 1 - 5 Carry One Mark Each

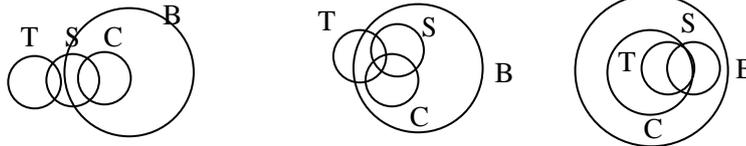
1. She has a sharp tongue and it can occasionally turn \_\_\_\_\_.  
 (A) hurtful (B) left (C) methodical (D) vital

**Key: (A)**

2. Some table are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusion can be deduced from the preceding sentences?  
 (i) At least one bench is a table  
 (ii) At least one shelf is a bench  
 (iii) At least one chair is a table  
 (iv) All benches are chairs  
 (A) only (i) (B) only (ii) (C) only (ii) and (iii) (D) only (iv)

**Key: (B)**

**Exp:**



3. 40% of deaths on city roads may be attributed to drunken driving. The number of degree needed to represent this as a slice of a pie chart is  
 (A) 120 (B) 144 (C) 160 (D) 212

**Key: (B)**

**Exp:** Given 40% of deaths on city roads are drunken driving

$$\text{w.k.t. in pie chart } 100\% \rightarrow 360^\circ \Rightarrow 1\% \rightarrow \left(\frac{360}{100}\right) \Rightarrow 40\% \rightarrow \frac{360}{100} \times 40 \Rightarrow 40\% \rightarrow 144^\circ$$

4. In the summer, water consumption is known to decrease overall by 25%. A water Board official states that in the summer household consumption decreases by 20%, while other consumption increases by 70%.

Which of the following statement is correct?

- (A) The ratio of household to other consumption is 8/17  
 (B) The ratio of household to other consumption is 1/17  
 (C) The ratio of household to other consumption is 17/8  
 (D) There are errors in the official's statement

**Key: (D)**

**Exp:** Let H is house hold consumption and P is the other consumption.

Given

$$H \times 0.8 + P \times 1.7 = (H + P) \times 0.75$$

$\Rightarrow$  Ratio is negative.

5. I \_\_\_\_\_ made arrangements had I \_\_\_\_\_ informed earlier.  
 (A) could have, been (B) would have, being  
 (C) had, have (D) had been, been

**Key: (A)**

**Q. No. 6- 10 Carry Two Marks Each**

6. "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective section, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".

Here, the word 'antagonistic' is closest in meaning to

- (A) impartial (B) argumentative (C) separated (D) hostile

**Key: (D)**

7. There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?  
 (A) 56 (B) 52 (C) 48 (D) 44

**Key:** (A)

**Exp:** No. of sub groups such that every sub group has at least one Indian

$$\begin{aligned}
 &= \underbrace{3_{C_1} + 3_{C_2} + 3_{C_3}}_{\text{Only Indians}} + \underbrace{3_{C_1} \times 3_{C_2} + 3_{C_1} + 3_{C_1} \times 3_{C_3}}_{\text{One Indian \& remaining Chinese}} \\
 &+ \underbrace{3_{C_2} \times 3_{C_1} + 3_{C_2} \times 3_{C_2} + 3_{C_2} \times 3_{C_3}}_{\text{2 Indians \& remaining Chinese}} + \underbrace{3_{C_3} \times 3_{C_1} + 3_{C_3} \times 3_{C_2} + 3_{C_3} \times 3_{C_3}}_{\text{3 Indians \& remaining Chinese}} \\
 &= 7 + 9 + 9 + 9 + 3 + 9 + 9 + 3 + 3 + 3 + 1 = 56.
 \end{aligned}$$

Alternate method

$$\text{Sub groups containing only Indians} = 3_{C_1} + 3_{C_2} + 3_{C_3} = 3 + 3 + 1 = 7$$

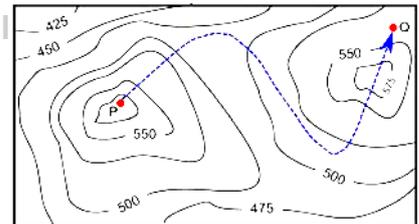
$$\text{Subgroups containing one Indian and rest Chinese} = 3_{C_1} [3_{C_1} + 3_{C_2} + 3_{C_3}] = 3[3 + 3 + 1] = 21$$

$$\text{Sub groups containing two Indian and remaining Chinese} = 3_{C_2} [3_{C_1} + 3_{C_2} + 3_{C_3}] = 21$$

$$\text{Sub groups containing three Indian and remaining Chinese} = 3_{C_3} [3_{C_1} + 3_{C_2} + 3_{C_3}] = 7$$

$$\therefore \text{Total no. of sub groups} = 7 + 21 + 21 + 7 = 56.$$

8. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.

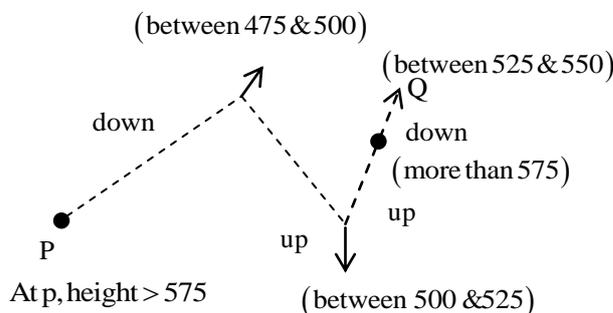


The path from P to Q is best described by

- (A) Up-Down-Up-Down  
 (B) Down-Up-Down-Up  
 (C) Down-Up-Down  
 (D) Up-Down-Up

**Key:** (C)

**Exp:** Down-up-Down

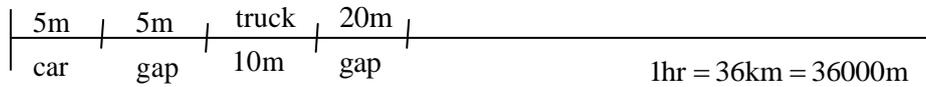


9. Trucks (10m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20 m after each truck and a gap of at least 15m after each car. Trucks and cars travel at a speed of 36 km/h. If cars and trucks go alternatively, what is the maximum number of vehicles that can use the bridge in one hour?  
 (A) 1440 (B) 1200 (C) 720 (D) 600

**Key:** (A)

**Exp:** Given speeds both car & Truck = 36 km/hour

They travel in 1 hr = 36 km = 36000 m.



∴ Maximum no. of vehicles than can use the bridge

$$\text{in 1 hour} = \frac{36000\text{m}}{50\text{m}} = 720\text{sets} = 720 \times 2 = 1440\text{ vehicles}$$

Alternate method

Length of truck + gap required = 10+20 = 30m

Length of car + gap required = 5+15 = 20m

Alternative pairs of Truck and car needs 30+ 20 = 50 m.

Let 'n' be the number of repetition of (Truck + car) in 1 hour (3600 sec).

Given speed = 36 km/hr = 10m/sec

$$\frac{50\text{m} \times n}{3600\text{sec}} = 36\text{km/hr}$$

$$\Rightarrow \frac{50n}{3600} \text{ m/sec} = 10\text{m/sec}$$

$$\Rightarrow n = \frac{36000}{50} = 720 (\text{Truck + car})$$

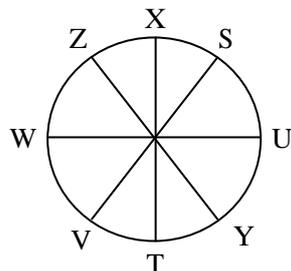
So, 720(Truck + car) passes = 720 × 2 = 1440 vehicles

10. S, T, U, V, W, X, Y and Z are seated around a circular table. T's neighbours are Y and V. Z is seated third to the left of T and second to the right of S. U's neighbours are S and Y; and T and W are not seated opposite each other. Who is third to the left of V?

- (A) X                      (B) W                      (C) U                      (D) T

**Key:** (A)

**Exp:** Following circular seating arrangement can be drawn.



Only one such arrangement can be drawn.

The person on third to the left of V is X.