Electronics and Communication Engineering

Q. No. 1 to 25 Carry One Mark Each

1. Consider the circuit shown in the figure.

![Circuit Diagram]

The Boolean expression \( F \) implemented by the circuit is

(A) \( \overline{XYZ} + XY + \overline{YZ} \)  
(B) \( \overline{XY}Z + XZ + \overline{YZ} \)  
(C) \( \overline{XY}Z + XY + \overline{YZ} \)  
(D) \( \overline{XYZ} + XY + \overline{YZ} \)

Key: (B)

Exp:  
\[ F = \overline{xy}z + z(\overline{xy}) \]
\[ F = \overline{xy}z + (x + \overline{y})z \]
\[ F = \overline{xy}z + xz + \overline{yz} \]

2. An LTI system with unit sample response \( h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4] \) is a

(A) Low – pass filter  (B) high – pass filter  (C) band – pass filter  (D) band – stop filter

Key: (C)

Exp:  
\[ h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4] \]

Obtain

\[ h[e^{-j\omega}] = 5 - 7e^{-j\omega} + 7e^{-3j\omega} - 5e^{-4j\omega} \]

At \( \omega = 0 \) and \( \frac{\pi}{2} \);

\[ h[e^{j0}] = 0 \]

For \( 0 < \omega < \frac{\pi}{2} \) at a frequency \( \omega_0 \) maximum value of \( h[e^{j\omega}] \) is obtained

Thus Ideal behaviour of \( h[n] \) is Band pass filter.
3. In the circuit shown, V is a sinusoidal voltage source. The current I is in phase with voltage V. The ratio of the amplitude of voltage across the capacitor to the amplitude of voltage across the resistor is __________.

Key: (0.19 to 0.21)

Exp: If I & V are in phase then the circuit is in resonance. At resonance,
\[
\frac{V_C}{V_R} = Q = \frac{1}{R} \sqrt{L/C} = \frac{1}{5 \sqrt{5}} = 0.2
\]

4. In a DRAM,
(A) periodic refreshing is not required
(B) information is stored in a capacitor
(C) information is stored in a latch
(D) both read and write operations can be performed simultaneously

Key: (B)

5. Consider an n-channel MOSFET having width W, length L, electron mobility in the channel \(\mu_n\) and oxide capacitance per unit area \(C_{ox}\). If gate-to-source voltage \(V_{GS}=0.7\) V, drain-to-source voltage \(V_{DS}=0.1\) V, \(\mu_n C_{ox} = 100\) \(\mu A/V^2\), threshold voltage \(V_{TH}=0.3\) V and \((W/L)=50\), then the transconductance \(g_m\) (in mA/V) is __________.

Key: (0.45 to 0.55)

Exp: Here, \(V_{DS} < V_{GS} = V_{PH}\), so n-channel MOSFET is working in linear region.

\[
I_D = \mu_n C_{ox} \frac{W}{L} \left( V_{GS} - V_{TH} \right) V_{DS} - \frac{V_{DS}^2}{2}
\]

So, transconductance \(g_m\) is in linear region and is given by

\[
g_m = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V_{GS}=\text{const}} = \left( \mu_n C_{ox} \right) \frac{W}{L} V_{DS} = 100 \times 10^{-6} \times 50 \times 0.1 = 5 \times 10^{-4} = 0.5 \text{ mA/V}
\]

6. Two conducting spheres S1 and S2 of radii a and b (b>a) respectively, are placed far apart and connected by a long, thin conducting wire, as shown in the figure.
For some charge placed on this structure, the potential and surface electric field on S1 are V_a and E_a, and that on S2 are V_b and E_b, respectively, which of the following is CORRECT?

(A) V_a = V_b and E_a < E_b
(B) V_a > V_b and E_a > E_b
(C) V_a = V_b and E_a > E_b
(D) V_a > V_b and E_a = E_b

Key: (C)

Exp: \( \Rightarrow \) bc, two spheres are joined with a conducting wire, the voltage on two spheres is same.
\( \Rightarrow V_a = V_b \Rightarrow \) The capacitance of sphere \( \propto \) radius
\[
\frac{C_a}{C_b} = \frac{a}{b}
\]
We know \( Q = CV \)
\[
\frac{Q_a}{Q_b} = \frac{C_a}{C_b} = \frac{a}{b}
\]
\[
\frac{E_a}{E_b} = \frac{1}{4\pi \varepsilon_0} \frac{a^2}{a} = b > 1
\]
\( E_a > E_b \)

7. For the circuit shown in the figure, P and Q are the inputs and Y is the output.

![Circuit Diagram](image)

The logic implemented by the circuit is

(A) XNOR  (B) XOR  (C) NOR  (D) OR

Key: (B)

8. An n-channel enhancement mode MOSFET is biased at \( V_{GS} > V_{TH} \) and \( V_{DS} > (V_{GS} - V_{TH}) \), where \( V_{GS} \) is the gate-to-source voltage, \( V_{DS} \) is the drain-to-source voltage and \( V_{TH} \) is the threshold voltage. Considering channel length modulation effect to be significant, the MOSFET behaves as a

(A) voltage source with zero output impedance
(B) voltage source with non-zero output impedance
(C) current source with finite output impedance
(D) current source with infinite output impedance

Key: (C)

Exp: If channel length modulation is considered and significant it means \( \lambda \neq 0 \)
\[ V_A \text{ (early voltage)} = \frac{1}{\lambda} \text{ and } r_c = \frac{V_A}{I_D} \]

If \( V_{AS} > V_{TH} \) and \( V_{DS} > (V_{DS} - V_{TH}) \) then it indicates that MOSFET is working in saturation region and it can be used as an amplifier. So it can act as current source with finite output impedance.

9. A connection is made consisting of resistance A in series with a parallel combination of resistances B and C. Three resistors of value 10Ω, 5Ω, 2Ω are provided. Consider all possible permutations of the given resistors into the positions A, B, C, and identify the configurations with maximum possible overall resistance, and also the ones with minimum possible overall resistance. The ratio of maximum to minimum values of the resistances (up to second decimal place) is ___________.

**Key:** (2.12 to 2.16)

**Exp:** The maximum resistance

\[
\begin{array}{c}
10\Omega \\
5\Omega \\
5\Omega \\
\end{array}
\]

\[ R_{\text{max}} = 11.428\Omega \]

The minimum resistance

\[
\begin{array}{c}
2\Omega \\
10\Omega \\
5\Omega \\
\end{array}
\]

\[ R_{\text{min}} = 5.333\Omega \]

\[ \frac{R_{\text{max}}}{R_{\text{min}}} = 2.14 \]

10. An npn bipolar junction transistor (BJT) is operating in the active region. If the reverse bias across the base – collector junction is increased, then

(A) the effective base width increases and common – emitter current gain increases

(B) the effective base width increases and common – emitter current gain decreases

(C) the effective base width decreases and common – emitter current gain increases

(D) the effective base width decreases and common – emitter current gain decreases

**Key:** (C)

**Exp:** If the reverse bias voltage across the base collector junction is increased, then their effective base width will decrease and collector current will increase, therefore their common-emitter current gain increases.
11. Consider the state space realization
\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & -9
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + \begin{bmatrix}
0 \\
45
\end{bmatrix} u(t),
\]
with the initial condition \[
\begin{bmatrix}
x_1(0) \\
x_2(0)
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]
where \(u(t)\) denotes the unit step function. The value of \(\lim_{t \to \infty} \sqrt{x_1^2(t) + x_2^2(t)}\) is ________.

Key: (4.99 to 5.01)

Exp:
\[
\begin{align*}
\dot{x}_1(t) &= 0 \quad \text{...(1)} \\
\dot{x}_2(t) &= -9x_2(t) + 45u(t) \quad \text{...(2)}
\end{align*}
\]
Apply L.T to above equation
\[
x_1(t) = 0 \quad \text{[because initial conditions are zero]}
\]
\[
Sx_2(s) - x_2(0) = -9X_2(s) + \frac{45}{s}
\]
\[
X_2(s)[s + 9] = \frac{45}{3}
\]
\[
X_2(s) = \frac{45}{s(s + 9)}
\]
\[
X_2(s) = \frac{5}{s + 9}
\]
\[
X_2(t) = 5u(t) - 5e^{-9t}u(t)
\]
\[
\text{It} \left| \sqrt{x_1^2(t) + x_2^2(t)} \right| = \text{It} \left| x_2(t) \right| = 5
\]

12. The rank of the matrix \[
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\]
is __________.

Key: (4 to 4)

Exp:
\[
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\]
\[
R_4 \rightarrow R_4 + R_1 \sim \begin{bmatrix}
1 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\]
13. A two – wire transmission line terminates in a television set. The VSWR measured on the line is 5.8. The percentage of power that is reflected from the television set is _____________

Key: (48.0 to 51.0)

Exp: Percentage of power reflected is \(\frac{VSWR - 1}{VSWR + 1}\) ×100

\[
\left| \Gamma \right| = \frac{VSWR - 1}{VSWR + 1} = \frac{5.8 - 1}{5.8 + 1} = \frac{4.8}{6.8} = 0.7058
\]

% Power reflected = \(\left| \Gamma \right|^2 \times 100\) = 49.82%

14. The input \(x(t)\) and the output \(y(t)\) of a continuous-time system are related as \(y(t) = \int_{-\infty}^{t} x(u)du\). The system is

(A) Linear and time-variant  
(B) Linear and time-invariant  
(C) Non-linear and time-variant  
(D) Non-linear and time-invariant

Key: (B)

Exp: Given input-output relationship describes integration over a fundamental period \(T\). The integration over one period is linear and time-invariant.
15. Which of the following statements is incorrect?
(A) Lead compensator is used to reduce the settling time.
(B) Lag compensator is used to reduce the steady state error.
(C) Lead compensator may increase the order of a system.
(D) Lag compensator always stabilizes an unstable system.

Key: (D)

Exp: The phase-lead controller adds zero and a pole, with the zero to the right of the pole, to the forward-path transfer function. The general effect is to add more damping to the closed-loop system. The rise time and settling time are reduced in general.

→ Reduces the steady state error
→ Reduces the speed of response (i.e. \( \xi \) decreases)
→ Increases the gain of original network without affecting stability
→ Permits the increases of gain if phase margin is acceptable
→ System becomes lesser stable
→ Reduces the effect of noise
→ Decrease the bandwidth

16. The residues of a function \( f(z) = \frac{1}{(z-4)(z+1)^2} \) are
(A) \(-\frac{1}{27}\) and \(-\frac{1}{125}\)
(B) \(\frac{1}{125}\) and \(-\frac{1}{125}\)
(C) \(-\frac{1}{27}\) and \(\frac{1}{5}\)
(D) \(\frac{1}{125}\) and \(\frac{1}{5}\)

Key: (B)

Exp: \( z = 4 \) is a pole of order ‘1’ (or) simple pole

\[
\text{Residue of } f(z) \text{ at } z=4 = \lim_{z \to 4} \left( z-4 \right) \frac{1}{(z-4)(z+1)^2} = \frac{1}{5(4)(16)} = \frac{1}{125}
\]

\( z = -1 \) is a pole of order ‘3’.

\[
\therefore \text{Res} f(z) = \frac{1}{(3-1)!} \lim_{z \to -1} \left\{ \frac{d^2}{dz^2} \left( z+1 \right)^3 \frac{1}{(z-4)(z+1)^2} \right\}
\]

\[
= \frac{1}{2} \lim_{z \to -1} \left\{ \frac{d^2}{dz^2} \left( \frac{1}{z-4} \right) \right\} = -\frac{1}{125}
\]

17. A sinusoidal message signal is converted to a PCM signal using a uniform quantizer. The required signal-to-quantization noise ratio (SQNR) at the output of the quantizer is 40dB. The minimum number of bits per sample needed to achieve the desired SQNR is _______

Key: (7 to 7)

Exp: For sinusoidal signal
\[(\text{SNR})_Q \text{ in dB} = 6.0n + 1.75\]

Given required \( (\text{SNR})_Q = 40 \text{dB} \)
\[ 6.0n + 1.75 \geq 40\text{dB} \]
\[ 6.0n \geq 40 - 11.75 \]
\[ n \geq \frac{40 - 11.75}{6.02} \]
\[ n = 7 \text{ (Since 'n' must be an integer)} \]

18. The general solution of the differential equation \( \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - 5y = 0 \) in terms of arbitrary constants \( K_1 \) and \( K_2 \) is

(A) \( K_1 e^{-1+\sqrt{5}}x + K_2 e^{-1-\sqrt{5}}x \)

(B) \( K_2 e^{-1+\sqrt{5}}x + K_2 e^{-1-\sqrt{5}}x \)

(C) \( K_1 e^{-2+\sqrt{5}}x + K_2 e^{-2-\sqrt{5}}x \)

(D) \( K_1 e^{-2+\sqrt{5}}x + K_2 e^{-2-\sqrt{5}}x \)

Key: (A)

Exp: \( D^2 + 2D - 5 = 0 \)
\[ \Rightarrow D = -1 \pm \sqrt{6} \text{ (roots are real and distinct)} \]
\[ \Rightarrow y = k_1 e^{-1+\sqrt{5}}x + k_2 e^{-1-\sqrt{5}}x \]

Where \( k_1, k_2 \) are arbitrary constants.

19. Which one of the following graphs shows the Shannon capacity (channel capacity) in bits of a memory less binary symmetric channel with crossover probability \( P \)?

(A) Capacity

(B) Capacity

(C) Capacity

(D) Capacity

Key: (C)

Exp: For memory less binary Symmetric channel

Channel capacity

\[ C = 1 - H(p) \]

\[ H(p) = p \log_2 \frac{1}{p} + (1 - p) \log_2 \frac{1}{1 + p} \]

\( p \rightarrow \text{Cross over probability} \)
\[ \Rightarrow C = 1 + p \log_2 p + (1 - p) \log_2 (1 - p) \]

At \( p = 0; \quad C = 1 \)
At \( p = 1; \quad C = 1 \)
At \( p = 1/2; \quad C = 0 \)
20. The output $V_0$ of the diode circuit shown in the figure is connected to an averaging DC voltmeter. The reading on the DC voltmeter in Volts, neglecting the voltage drop across the diode, is __________.

![Diode Circuit Diagram]

Key: (3.15 to 3.21)

Exp: $V_0 = \frac{V_m}{\pi} = \frac{10}{\pi} \approx 3.1847V$

21. Consider the random process $X(t) = U + Vt$, where $U$ is a zero-mean Gaussian random variable and $V$ is a random variable uniformly distributed between 0 and 2. Assume that $U$ and $V$ are statistically independent. The mean value of the random process at $t = 2$ is __________

Key: (2)

Exp: Given $x(t) = U + Vt$

$x(2) = U + 2V$

$E[x(2)] = E[U + 2V] = E[U] + 2E[V] = 0 + 2 \times 1 = 2$

22. For the system shown in the figure, $Y(s) / X(s) =$ __________

![System Diagram]

Key: (0.95 to 1.05)

Exp: $\frac{Y(s)}{X(s)} = \frac{2 + s}{1 + s} = 1$

23. The smaller angle (in degrees) between the planes $x + y + z = 1$ and $2x - y + 2z = 0$ is __________.

Key: (54.0 to 55.0)

Exp: $x + y + 2 = 1$

$2x - y + 2z = 0$

We have angle between two planes

$a_1x + b_1y + c_1y + d_1 = 0$

$a_2x + b_2y + c_2y + d_2 = 0$

is $\cos \theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\Rightarrow \cos \theta = \frac{|2 - 1 + 2|}{\sqrt{1 + 1 + 1} \sqrt{4 + 1 + 4}} = \frac{3}{\sqrt{3} \sqrt{9}} = \frac{1}{\sqrt{3}} \Rightarrow \theta \approx 54.73$
24. Consider the circuit shown in the figure. Assume base-to-emitter voltage $V_{BE}=0.8 \, \text{V}$ and common base current gain ($\alpha$) of the transistor is unity.

![Circuit Diagram]

The value of the collector-to-emitter voltage $V_{CE}$ (in volt) is _______.

**Key:** (5.5 to 6.5)

**Exp:** Given $V_{BE} = 0.8\, \text{V}; \alpha = 1$

As $\alpha = 1; \beta$ is very large

So $I_E = I_C$

$V_B = \frac{18 \times 16}{60} = 4.8 \, \text{V}$

$I_C = \frac{4.8 - 0.8}{2 \times 10^{-3}} = 2 \, \text{mA}$

$V_{CE} = 18 - 6 \times 10^{-1} \times 2 \times 10^{-3}$

$= 18 - 12 = 6 \, \text{V}$

25. In the figure, D1 is a real silicon pn junction diode with a drop of 0.7V under forward bias condition and D2 is a zener diode with breakdown voltage of -6.8 V. The input $V_{in}(t)$ is a periodic square wave of period $T$, whose one period is shown in the figure.

![Circuit Diagram]

Assuming $10\tau \ll T$, where $\tau$ is the time constant of the circuit, the maximum and minimum values of the output waveform are respectively,

(A) 7.5 V and $-20.5\, \text{V}$

(B) 6.1 V and $-21.9\, \text{V}$

(C) 7.5 V and $-21.2\, \text{V}$

(D) 6.1 V and $-22.6\, \text{V}$

**Key:** (A)

**Exp:** When $V_i = 14\, \text{V}$, the equivalent circuit is
Q. No. 26 to 55 Carry Two Marks Each

26. If the vector function \( \vec{F} = \hat{a}_i (3y - k z) + \hat{a}_j (k x - 2z) - \hat{a}_k (k y + z) \) is irrotational, then the values of the constants \( k_1, k_2 \) and \( k_3 \) respectively, are
   (A) 0.3, –2.5, 0.5  
   (B) 0.0, 3.0, 2.0  
   (C) 0.3, 0.33, 0.5  
   (D) 4.0, 3.0, 2.0

Key: (B)

Exp: curl \( \vec{F} = 0 \)

\[
\begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
3y - k z & k x - 2z & -k y - z
\end{vmatrix} = 0
\]

\[
\Rightarrow (k_1 + 2) - (0 + k_1) + k_2 (k_2 - 3) = 0
\]

\[
\Rightarrow k_1 = 0, k_2 = 3, k_3 = 2
\]

27. The un-modulated carrier power in an AM transmitter is 5kW. This carrier is modulated by a sinusoidal modulating signal. The maximum percentage of modulation is 50%. If it is reduced to 40%, then the maximum un-modulated carrier power (in kW) that can be used without overloading the transmitter is ___________

Key: (5.19 to 5.23)

Exp: Total power when \( \mu = 50\% \) is

\[
P_t = P_c \left[ 1 + \frac{\mu^2}{2} \right]
\]
\[ P_T = 5 \left[ 1 + \frac{(0.5)^2}{2} \right] = 5[1 + 0.125] = 5[1.125] \]
\[ P_T = 5.625 \]
When \( \mu = 40\% \)
Total power remains 5.625
\[ \Rightarrow 5.625 = P_c \left[ 1 + \frac{(0.4)^2}{2} \right] \Rightarrow 5.625 = P_c [1 + 0.08] \]
\[ P_c = 5.22 \]

28. Consider an LTI system with magnitude response
\[ |H(f)| = \begin{cases} 
1 - \frac{|f|}{20}, & |f| \leq 20 \\
0, & |f| > 20 
\end{cases} \]
And phase response \( \text{Arg} \{H(f)\} = -2f \).
If the input to the system is
\[ x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right). \]
Then the average power of the output signal \( y(t) \) is _________.

Key: \( (7.95 \text{ to } 8.05) \)

Exp: Consider an LTI system with magnitude response
\[ |H(f)| = \begin{cases} 
1 - \frac{|f|}{20}, & |f| \leq 20 \\
0, & |f| > 20 
\end{cases} \]
And phase response \( \text{Arg} \{H(f)\} = -2f \).
If the input to the system is
\[ x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right). \]
Then the average power of the output signal \( y(t) \) is _________.

28. Obtain \( X(f) \) for the given \( x(t) \)
\[ |X(f)| = 4\left[\delta(f - 10) + \delta(f + 10)\right] + 8\left[\delta(f - 20) - \delta(f + 20)\right] \\
12\left[\delta(f - 40) + \delta(f + 40)\right] \]
\[ y(t) = 4\cos 2\pi t \]

Thus max power \( \frac{16}{2} = 8 \)

29. A MOS capacitor is fabricated on p-type Si (silicon) where the metal work function is 4.1 eV and electron affinity of Si is 4.0 eV. \( E_C - F_F = 0.9 \) eV, where \( E_C \) and \( F_F \) are the conduction band minimum and the Fermi energy levels of Si, respectively. Oxide \( \varepsilon_r = 3.9, \varepsilon_0 = 8.85 \times 10^{-12} \text{F/cm} \). oxide thickness \( t_{ox} = 0.1 \mu m \) and electronic charge \( q = 1.6 \times 10^{-19} \text{C} \). If the measured flat band voltage of the capacitor is \(-1V\), then the magnitude of the fixed charge at the oxide-semiconductor interface, in nC/cm\(^2\), is __________.

Key: (6.85 to 6.95)

Exp:

\[ V_{FB} = q\phi_{MS} = \frac{Q_f}{C_{ox}} \]

\[ q\phi_{MS} = q\phi_M - q\phi_S \]

\[ q\phi_M - q\phi_S (E_c - E_F) = 4.1 - 4.0 - 0.9 = 0.1 - 0.9 = -0.8 \text{eV} \]

\[ \phi_{MS} = \frac{-0.8 \times q \times lV}{q} = -0.8 \text{V} \]

\[ C_{ox} = \frac{E_{ox}}{t_{ox}} = 34.5 \times 10^{-9} \text{F/cm}^2 \]

\[ -1 = -0.8 - \frac{Q_f}{34.5 \times 10^{-9}} \]

\[ -0.2 = - \frac{Q_f}{34.5 \times 10^{-9}} \]

\[ Q_f = 6.9 \text{nC/cm}^2 \]

30. An electron \((q_1)\) is moving in free space with velocity \(10^5 \text{ m/s}\) towards a stationary electron \((q_2)\) far away. The closest distance that this moving electron gets to the stationary electron before the repulsive force diverts its path is __________ \(\times 10^8\)m.

[Given, mass of electron \( m = 9.11 \times 10^{-31} \text{kg} \), charge of electron \( e = -1.6 \times 10^{-19} \text{C} \), and permittivity \( \varepsilon_0 = (1/36\pi) \times 10^{-9} \text{F/m} \)]

Key: (4.55 to 5.55)

Exp:

Work done due to field and external agent must be zero

\[ qV = \frac{1}{2} MV^2 \]

\[ \Rightarrow -1.6 \times 10^{-19} \times \frac{1.6 \times 10^{-19}}{4\pi \varepsilon_0 \gamma} = \frac{1}{2} m \times (10^5)^2 \]

\[ \Rightarrow \gamma = 5.058 \times 10^{-8} \text{ m} \]
31. The values of the integrals
\[ \int_{0}^{1} \left( \int_{0}^{x-y} \frac{x-y}{x+y} \, dy \right) \, dx \text{ and } \int_{0}^{1} \left( \int_{0}^{x-y} \frac{y}{x+y} \, dx \right) \, dy \] are
(A) same and equal to 0.5  (B) same and equal to -0.5
(C) 0.5 and -0.5, respectively  (D) -0.5 and 0.5, respectively

Key: (C)

Exp: To find \( \int_{0}^{1} \left( \int_{0}^{x-y} \frac{x-y}{x+y} \, dy \right) \, dx \)

Consider \( \int_{0}^{x-y} \frac{x-y}{x+y} \, dy = x \frac{1}{2} \left[ \frac{1}{(x+y)^2} - \frac{1}{x^2} \right] \)

\[ = \frac{x(x+y)^{-3+1}}{-3+1} - \int_{0}^{x} \frac{1}{(x+y)^2} \, dy + x \frac{1}{2} \left[ \frac{1}{(x+1)^2} - \frac{1}{x^2} \right] \]

\[ = -\frac{x}{2} \left[ \frac{1}{(x+1)^2} - \frac{1}{x^2} \right] - \int_{0}^{x} \frac{1}{(x+y)^2} \, dy + x \frac{1}{2} \left[ \frac{1}{(x+1)^2} - \frac{1}{x^2} \right] \]

\[ = -\frac{x}{2} \left[ \frac{1}{(x+1)^2} - \frac{1}{x^2} \right] + \frac{1}{x+1} + \frac{1}{x} \]

\[ = \frac{1}{(x+1)^2} \]

\[ \therefore \int_{0}^{1} \left( \int_{0}^{x-y} \frac{x-y}{x+y} \, dy \right) \, dx = \int_{0}^{1} \left( \frac{1}{(x+1)^2} \right) \, dx = -\frac{1}{x+1} \bigg|_{0}^{1} = \frac{1}{2} \]

Similarly \( \int_{0}^{1} \left( \int_{0}^{x-y} \frac{y}{x+y} \, dx \right) \, dy = -\frac{1}{2} \)

32. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is _________.

Key: (2.4 to 2.6)

Exp: Let ‘X’ is a random variable which takes number of attempts

Given probability of any attempts to be successful,

\[ p = 40\% = \frac{40}{100} = \frac{2}{5}, \quad q = 1 - \frac{2}{5} = \frac{3}{5} \]

\[
\begin{array}{c|c|c|c|c|c}
X & 1 & 2 & 3 & 4 & -
\hline
p(X) & \frac{2}{5} & \left(\frac{3}{5}\right) \times \left(\frac{2}{5}\right) & \left(\frac{3}{5}\times\frac{3}{5}\right) \times \left(\frac{2}{5}\right) & \left(\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\right) \times \left(\frac{2}{5}\right) & -
\end{array}
\]
33. Figure I shows a 4-bits ripple carry adder realized using full adders and Figure II shows the circuit of a full-adder (FA). The propagation delay of the XOR, AND and OR gates in Figure II are 20 ns, 15 ns and 10 ns respectively. Assume all the inputs to the 4-bit adder are initially reset to 0.

At \( t=0 \), the inputs to the 4-bit adder are changed to \( X_3X_2X_1X_0 = 1100, Y_3Y_2Y_1Y_0 = 0100 \) and \( Z_n = 1 \).

The output of the ripple carry adder will be stable at \( t \) (in ns) = ___________

Key: (70.0 to 70.0)

34. The permittivity of water at optical frequencies is \( 1.75 \varepsilon_0 \). It is found that an isotropic light source at a distance \( d \) under water forms an illuminated circular area of radius 5m, as shown in the figure. The critical angle is \( \theta_c \).

The value of \( d \) (in meter) is ____________

Key: (4.2 to 4.4)

Exp: 

\[ \theta_c = \sin^{-1}\left(\frac{\varepsilon_r}{\sqrt{\varepsilon_t}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{1.75}}\right) = 49.106 \]

\[ \Rightarrow \tan \theta_c = \frac{5}{d} \Rightarrow d = \frac{5}{\tan \theta_c} = 4.33m \]
35. A unity feedback control system is characterized by the open-loop transfer function

\[ G(s) = \frac{10K(s+2)}{s^3 + 3s^2 + 10} \]

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figures below.

If \( 0 < K < 1 \), then the number of poles of the closed-loop transfer function that lie in the right half of the \( s \)-plane is

(A) 0  (B) 1  (C) 2  (D) 3

**Key:**  (C)

**Exp:**

\( N=0 \), Because \( 0 < L < 1 \)

There are no encircles around \((Y, 0)\)

And

\[ G(S) = \frac{10K(S+2)}{S^3 + 3S^2 + 10} = \frac{10K(S+2)}{(S+3.72)(S-(0.31\pm1.598i))} \]

So,

\[ P = 2 \]

\[ N = P - Z \]

\[ Z = 2 \]

OR

\[ C.E = S^3 + 3S^2 + 10KS + 20K + 10 \]

If stable \( 30K > 20K+10 \)

\( K > 1 \)

Here, in the question asking \( 0 < K < 1 \)

So, System is unstable

36. The signal \( x(t) = \sin(14000\pi t) \), where \( t \) is in seconds is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal low pass filter with frequency response \( H(f) \) as follows:

\[ H(f) = \begin{cases} 1, & |f| \leq 12kHz \\ 0, & |f| > 12kHz. \end{cases} \]

What is the number of sinusoids in the output and their frequencies in kHz?

(A) Number = 1, frequency = 7  (B) Number = 3, frequencies = 2, 7, 11  
(C) Number = 2, frequencies = 2, 7  (D) Number = 2, frequencies = 2, 11

**Key:**  (B)
Exp: Given input signal \( x(t) = \sin(1400\pi t) \)

**Input signal spectrum**

<table>
<thead>
<tr>
<th>( f ) (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
</tr>
<tr>
<td>-2</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

Sampled signal spectrum is the spectrum of \( X(f) \) which repeats with integer multiples of 9 kHz.

**Sampled signal spectrum:**

<table>
<thead>
<tr>
<th>( f ) (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16</td>
</tr>
<tr>
<td>-11</td>
</tr>
<tr>
<td>-9</td>
</tr>
<tr>
<td>-7</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>16</td>
</tr>
</tbody>
</table>

The sampled signal spectrum is passed through a LPF of cutoff frequency 12 KHz. Thus the filtered out sinusoids are of 2 KHz 7 KHz and 11 KHz frequency.

37. A unity feedback control system is characterized by the open-loop transfer function

\[
G(s) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1}
\]

The value of \( k \) for which the system oscillates at 2 rad/s is ________.

**Key:** (0.74 to 0.76)

Exp:

\[
G(s) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1}
\]

\( \omega = 2 \text{ rad/sec} \)

\( K = ?? \)

\[
1 + G(s)H(s) = 0
\]

\[
1 + \frac{2(k+1)}{s^3 + ks^2 + 2s + 1} \Rightarrow s^3 + ks^2 + 4s + 3 = 0
\]

\[
s^3 \hspace{1cm} 1 \hspace{1cm} 4
\]

\[
s^2 \hspace{1cm} k \hspace{1cm} 3
\]

\[
s^1 \hspace{1cm} \frac{4k-3}{k} \hspace{1cm} 0
\]

\[
s^0 \hspace{1cm} \frac{3}{k}
\]

For marginal stable

\[
\frac{4k-3}{k} = 0 \Rightarrow k = \frac{3}{4} = 0.75
\]

Cross check
Take auxiliary equation
\[ ks^2 + 3 = 0 \]
\[ \frac{3}{4} s^2 + 3 = 0 \]
\[ s = \pm j2 \]
\[ \omega = 2 \text{ rad/sec} \]
\[ k = \frac{3}{4} \]

38. Consider the circuit shown in the figure.

![Circuit Diagram](image)

The Thevenin equivalent resistance (in \( \Omega \)) across P – Q is ______.

**Key:** (-1.01 to -0.99)

**Exp:** To find \( R_{th} = \frac{V}{I} \)

Here \( i_0 = \frac{V}{I} = V \)
Nodal at V
\[ \frac{V - 3i_0}{I} + \frac{V}{1} - 1 = 0 \]
\[ V - 3V + V - 1 = 0 \]
\( R_{th} = -1\Omega \)

39. The transfer function of a causal LTI system is \( H(s) = \frac{1}{s} \). If the input to the system is \( x(t) = \frac{\sin(t)}{\pi t} u(t) \), where \( u(t) \) is a unit step function, the system output \( y(t) \) as \( t \to \infty \) is ______.

**Key:** (0.45 to 0.55)

**Exp:** Given \( x(t) = \frac{\sin t u(t)}{\pi t} \)

By using frequency integration property,
\[ \frac{x(t)}{t} \leftarrow \int_{-\infty}^{t} X_1(u)du \]
Consider \( x_1(t) = \sin t u(t) \leftarrow \int_{-\infty}^{t} \frac{1}{s^2 + 1} = X_1(s) \)
\[ \therefore \int_{-\infty}^{t} \left( \frac{1}{u^2 + 1} \right) du = \frac{\pi}{2} \tan^{-1} (s) \]
\[ L\left[ x(t) \right] = \frac{1}{\pi} \left[ \frac{\pi}{2} - \tan^{-1}(s) \right] = X(s) \]

\[ \therefore Y(s) = X(s)H(j) = \frac{1}{2s} - \frac{1}{\pi s} \tan^{-1}s \]

By using final value theorem,

\[ \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \left[ \frac{1}{2} - \frac{1}{2} \tan^{-1}(s) \right] = \frac{1}{2} \]

40. An integral I over a counter clock wise circle C is given by

\[ I = \oint_C \frac{z^2 - 1}{z^2 + 1} e^z dz. \]

If \( C \) is defined as \( |z| = 3 \), then the value of I is

(A) \(-\pi \sin(1)\) \quad (B) \(-2\pi \sin(1)\) \quad (C) \(-3\pi \sin(1)\) \quad (D) \(-4\pi \sin(1)\)

Key: (D)

Exp: \[ I = \oint_C \left( \frac{z^2 - 1}{z^2 + 1} \right) e^z dz \]

Consider \( f(z) = e^z \left( \frac{z^2 - 1}{z^2 + 1} \right) = e^z \left( \frac{z^2 - 1}{(z+1)(z-i)} \right) \)

\[ \Rightarrow z = \pm i \text{ are simple poles of } f(z) \text{ which lie inside } |z| = 3 \]

Residue of \( f(z) \) at \( z = i = \lim_{z \to i} (z - i) \left( \frac{z^2 - 1}{(z+1)(z-i)} \right) = ie^i \]

& Residue of \( f(z) \) at \( z = -i = \lim_{z \to -i} (z + i) \left( \frac{z^2 - 1}{(z+1)(z-i)} \right) = -ie^{-i} \]

\[ \therefore \text{ By residue theorem, } I = \oint_C \left( \frac{z^2 - 1}{z^2 + 1} \right) e^z dz = 2\pi i \left( ie^i - ie^{-i} \right) = -2\pi \left( e^i - e^{-i} \right) \]

\[ = -4\pi i \left( \frac{e^i - e^{-i}}{2i} \right) = -4\pi \sin(1) \]

41. Consider a binary memory less channel characterized by the transition probability diagram shown in the figure.

The channel is

(A) Lossless \quad (B) Noiseless \quad (C) Useless \quad (D) Deterministic

Key: (C)
Exp: It is a useless channel as MAP criteria cannot decide anything on receiving ‘0’ we cannot decide what is transmitted.

42. An abrupt pn junction (located at $x = 0$) is uniformly doped on both p and n sides. The width of the depletion region is $W$ and the electric field variation in the $x$-direction is $E(x)$. Which of the following figures represents the electric field profile near the pn junction?

(A) ![Diagram A]

(B) ![Diagram B]

(C) ![Diagram C]

(D) ![Diagram D]

Key: (A)

Exp: If left side is p-region and right side is n-region then electric field triangle will be down warded and if the left side is n-region and right side is p-region, then electric field triangle will be upward.

43. A second – order LTI system is described by the following state equations,

$$\frac{d}{dt} x_1(t) - x_2(t) = 0$$
$$\frac{d}{dt} x_2(t) + 2x_1(t) + 3x_2(t) = r(t)$$

Where $x_1(t)$ and $x_2(t)$ are the two state variables and $r(t)$ denotes the input. The output $c(t) = x_1(t)$. The system is.

(A) Undamped (oscillatory)  
(B) Under damped  
(C) Critically damped  
(D) Over damped

Key: (D)

Exp:

$$\dot{x}_1(t) = x_2(t)$$

$$sX_1(s) = X_2(s) \rightarrow (1)$$

$$\dot{x}_2(t) + 2x_1(t) + 3x_2(t) = r(t)$$

$$sX_2(s) + 2X_1(s) + 3X_2(s) = R(s)$$

$$\left[ s^2 + 2s \right] X_1(s) = R(s)$$

$$C(s) = X_1(s) = \frac{R(s)}{s^2 + 2s + 2}$$

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1)(s+2)}$$

system is over damped
44. Consider the parallel combination of two LTI systems shown in the figure.

![Parallel LTI Systems Diagram]

The impulse responses of the systems are

\[ h_1(t) = 2\delta(t+2) - 3\delta(t+1) \]
\[ h_2(t) = \delta(t-2) \]

If the input \( x(t) \) is a unit step signal, then the energy of \( y(t) \) is __________.

**Key:** (7.0 to 7.0)

**Exp:** Since \( h_1(t) \) and \( h_2(t) \) are connected in parallel the resultant system can be given as follows.

\[ y(t) = x(t) * [h_1(t) + h_2(t)] \]

\[ y(t) = 2u(t+2) - 3u(t+1) + u(t-2) \]

\[ \text{Energy of } y(t) = \int_{-2}^{1} (2)^2 dt + \int_{-2}^{2} (-1)^2 dt = 4[1] + 1[3] = 7 \]

45. Assuming that transistors \( M_1 \) and \( M_2 \) are identical and have a threshold voltage of 1V, the state of transistors \( M_1 \) and \( M_2 \) are respectively.

(A) Saturation, Saturation
(B) Linear, Linear
(C) Linear, Saturation
(D) Saturation, Linear

**Key:** (C)

**Exp:** If \( V_D \geq V_G - V_{\text{TH}} \), then transistor is working in saturation region.

So, For \( M_1 \) transistor

\[ V_{D_1} > V_G - V_{\text{TH}} \]
\[ 3V > (2.5 - 1)V \]

Assume that \( M_1 \) is working in saturation, so that
I_{D1} = I_{D2}
V_{GS1} - V_{TH} = V_{GS2} - V_{TH}
2V = V_{G2} - V_{S2}
= 2.5V - V_{S2}
\therefore V_{S2} = V_{D1} = 0.5V.

Now, for M_1, transistor to work in saturation \( V_{D1} \geq V_{GS1} - V_{TH} \), but it is not satisfied by M_1 transistor and \( V_{GS1} > V_{TH} \), so, transistor M_1 is ON but working in linear region.

46. A programmable logic array (PLA) is shown in the figure.

The Boolean function \( F \) implemented is

(A) \( \bar{P} \bar{Q} R + \bar{P} Q R + P \bar{Q} R \)

(B) \( (\bar{P} + \bar{Q} + R)(\bar{P} + Q + R)(P + Q + R) \)

(C) \( \bar{P} \bar{Q} R + \bar{P} Q R + P \bar{Q} R \)

(D) \( (\bar{P} + \bar{Q} + R)(\bar{P} + Q + R)(P + Q + R) \)

Key: (C)

Exp: \( \bar{P} \bar{Q} R + \bar{P} Q R + P \bar{Q} R \)

47. A modulating signal given by \( x(t) = 5 \sin \left( 4\pi \times 10^4 t - 10\pi \cos \left( 2\pi \times 10^4 t \right) \right) V \) is fed to a phase modulator with phase deviation constant \( k_p = 0.5 \text{ rad/V} \). If the carrier frequency is 20 kHz, the instantaneous frequency \( f_i(t) \) at \( t = 0.5 \text{ ms} \) is \( 69.9 \text{ to } 70.1 \text{ kHz} \)

Exp: \( x(t) = 5 \sin \left( 4\pi \times 10^4 t - 10\pi \cos \left( 2\pi \times 10^4 t \right) \right) \)

Transform theorem frequency

\[
f_i(t) = f_c + \frac{1}{2\pi} k_p \frac{d}{dt} m(t)
\]

\[
\frac{d}{dt} m(t) = 5 \cos \left( 4\pi \times 10^4 t - 10\pi \cos \left( 2\pi \times 10^4 t \right) \right) \left( 4\pi \times 10^3 + 10\sin \left( 2\pi \times 10^4 t \right) \right)\]

\[
\frac{d}{dt} m(t) \bigg|_{t=0.5} = 5 \cos \left( 2\pi + 10\pi \right) \times \left( 4\pi \times 10^3 + 0 \right) = 20\pi \times 10^3 \cos \left( 12\pi \right) = 20\pi \times 10^3
\]

\[
f_i(t) = 20 + \frac{1}{2\pi} \times 5 \times 20\pi = 70 \text{ kHz}
\]
48. The minimum value of the function $f(x) = \frac{1}{3}x(x^2 - 3)$ in the interval $-100 \leq x \leq 100$ occurs at $x =$ ________.

Key: (-100.01 to -99.99)

Exp: 

$$f(x) = \frac{1}{3}x(x^2 - 3) = \frac{x^3}{3} - x$$

$$f'(x) = \frac{3x^2}{3} - 1 = x^2 - 1$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow x = \pm 1$$

$$f''(x) = 2x$$

$$f''(1) = 2 > 0 \Rightarrow \text{at } x = 1, f(x) \text{ has local minimum.}$$

$$f''(-1) = -2 < 0 \Rightarrow \text{at } x = -1, f(x) \text{ has local maximum.}$$

For $x = 1$, local minimum value $= f(1) = \frac{1}{3} - 1 = \frac{-2}{3}$

Finding $f(-100) = -333433.33$

$$f(100) = 333233.33$$

($\because$ $x = 100, -100$ are end points of interval)

$\therefore$ Minimum occurs at $x = -100$

49. The switch in the circuit, shown in the figure, was open for a long time and is closed at $t = 0$. The current $i(t)$ (in ampere) at $t = 0.5$ seconds is ________
\[
\tau = \frac{L}{R} = \frac{2.5}{5} = \frac{1}{2} \Rightarrow i_L(\infty) = 0
\]

\[i_L(t) = 5e^{-2t}\]

\[i(t) = 10 - 5e^{-2t}\]

At \(t = 0.5s\)

\[i(0.5) = 10 - \frac{5}{e} = 8.16A\]

50. In the voltage reference circuit shown in the figure, the op-amp is ideal and the transistors \(Q_1\), \(Q_2\), \ldots, \(Q_{32}\) are identical in all respects and have infinitely large values of common – emitter current the relation \(I_C = I_S \exp \left(\frac{V_{BE}}{V_T}\right)\), where \(I_S\) is the saturation current. Assume that the voltage \(V_P\) shown in the figure is 0.7 V and the thermal voltage \(V_T = 26mV\).

The output voltage \(V_{out}\) (in volts) is __________.

**Key: (1.1 to 1.2)**

**Exp: KCL at node ‘a’**

\[
\frac{V_o - V_c}{20} = \frac{V_c - 0.7}{5}
\]

\[V_c - V_i = 4V_c - 2.8\]

\[V_o = 5V_c - 2.8\]

Now, \(I_a e^{V_s/V_T} = 31I_a e^{V_s/V_T}\)

\[
\frac{V_c}{V_f} = \epsilon n 31 + \frac{V_s}{V_f}
\]

\[
\frac{V_c - V_s}{V_f} = \epsilon n 31
\]

\[
\Rightarrow V_c = 0.789V
\]

From equation (i) \(V_o = 5 \times 0.789 - 2.8 = 1.145V\)
51. The state diagram of a finite state machine (FSM) designed to detect an overlapping sequence of three bits is shown in the figure. The FSM has an input ‘In’ and an output ‘Out’. The initial state of the FSM is $S_0$.

If the input sequence is 10101101001101, starting with the left-most bit, then the number times ‘Out’ will be 1 is __________.

**Key:** (4 to 4)

**Exp:** From the state diagram, let us obtain the transition of states and out when IN channel.

Initial state is

So, the input sea is 10101101001101

When

IN = 1 then $S_0 \rightarrow S_1$, with out = 0

IN = 0 then $S_1 \rightarrow S_2$, with out = 0

IN = 1 then $S_2 \rightarrow S_3$, with out = 1

IN = 0 then $S_3 \rightarrow S_0$, with out = 0

IN = 1, then $S_2 \rightarrow S_3$, with out = 1

IN = 0, then $S_1 \rightarrow S_2$, with out = 0

IN = 1, then $S_3 \rightarrow S_1$, with out = 0

IN = 0, then $S_1 \rightarrow S_2$, with out = 0

IN = 1, then $S_0 \rightarrow S_1$, with out = 0

IN = 1, then $S_1 \rightarrow S_2$, with out = 0

IN = 0, then $S_2 \rightarrow S_3$, with out = 1

IN = 1, then $S_2 \rightarrow S_3$, with out = 1

→ The ticketed mark now corresponding to output = 1.

So output will be 1 ‘4’ times.

52. Standard air – filled rectangular waveguides of dimensions $a = 2.29$ cm and $b= 1.02$ cm are designed for radar applications. It is desired that these waveguides operate only in the dominant $TE_{10}$ mode but not higher than 95% of the next higher cutoff frequency. The range of the allowable operating frequency $f$ is.

(A) $8.19$ GHz $\leq f \leq 13.1$ GHz
(B) $8.19$ GHz $\leq f \leq 12.45$ GHz
(C) $6.55$ GHz $\leq f \leq 13.1$ GHz
(D) $1.64$ GHz $\leq f \leq 10.24$ GHz

**Key:** (B)
Exp: Cut off frequency of TE_{10} is \( f_c = \frac{c}{2a} \times \frac{1}{2} \times \frac{1}{2.29 \times 10^{-2}} = 65.5 \times 10^6 \text{Hz} \)

Since, \( b \frac{a}{2} \Rightarrow \text{next higher mode is TE}_{20} \)

\[ f_{c|TE_{20}} = \frac{c}{2a} \times \frac{2}{2} = 13.1 \text{GHz} \]

\[ f \leq 0.95 \times 13.1 = 12.45 \text{GHz} \]

53. For a particular intensity of incident light on a silicon pn junction solar cell, the photocurrent density (\( J_L \)) is 2.5 mA/cm\(^2\) and the open-circuit voltage (\( V_{oc} \)) is 0.451 V. Consider thermal voltage (\( V_T \)) to be 25mV. If the intensity of the incident light is increased by 20 times, assuming that the temperature remains unchanged. \( V_{oc} \) (in volts) will be ______.

Key: (0.51 to 0.54)

Exp: \[ J_s = \frac{J_L}{\left( e^{V_{oc}/V_T} \right) - 1} = \frac{2.5 \times 10^{-3}}{e^{0.025} - 1} = 3.6 \times 10^{-11} \text{A/cm}^2 \]

Now if the intensity of the light is increased by 20 times it means their photocurrent will also increased by 20 times.

\[ V_{oc} = \frac{K T}{q} \log \left( \frac{J_s}{J_s + 1} \right) \]

\[ = 25 \times 10^{-3} \log \left( \frac{20 \times 2.5 \times 10^{-3}}{3.6 \times 10^{-11}} + 1 \right) \]

\[ = 0.5262 \text{Volt} \]

54. In the circuit shown, transistors Q\(_1\) and Q\(_2\) are biased at a collector current of 2.6mA. Assuming that transistor current gains are sufficiently large to assume collector current equal to emitter current and thermal voltage of 26 mV, the magnitude of voltage gain \( V_0/V_s \) in the mid-band frequency range is ___________ (up to second decimal place).

Key: (49.0 to 51.0)

Exp: a.c equivalent circuit for the given figure
55. Two n-channel MOSFETs, T1 and T2, are identical in all respects except that the width of T2 is double that of T1. Both the transistors are biased in the saturation region of operation, but the gate overdrive voltage \( V_{GS} - V_{TH} \) of T2 is double that of T1, where \( V_{GS} \) and \( V_{TH} \) are the gate – to – source voltage and threshold voltage of the transistors, respectively. If the drain current and transconductance of T1 are \( I_{D1} \) and \( g_{m1} \) respectively, the corresponding values of these two parameters for T2 are

(A) \( 8I_{D1} \) and \( 2g_{m1} \)  
(B) \( 8I_{D1} \) and \( 4g_{m1} \)  
(C) \( 4I_{D1} \) and \( 4g_{m1} \)  
(D) \( 4I_{D1} \) and \( 2g_{m1} \)

Key: (B)

Exp: Drain current in saturation is

\[
I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2
\]

For transistor T1

\[
I_D = I_{D1} \quad \text{and} \quad g_m = g_{m1} = \frac{\partial I_{D1}}{\partial V_{GS}} = \mu_n C_{ox} \frac{V_{GS} - V_{TH}}{L}
\]

For transistor T2

\[
W_2 = 2W_1 = 2W
\]

\[
(V_{GS} - V_{TH})_2 = 2(V_{GS} - V_{TH})_1 = 2(V_{GS} - V_{TH})
\]

\[
I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{2W}{L} [2(V_{GS} - V_{TH})]^2 = 8I_{D1}
\]

\[
g_{m2} = \frac{\partial I_{D2}}{\partial V_{GS2}} = \mu_n C_{ox} \frac{2W}{L} \times 2(V_{GS} - V_{TH}) = 4g_{m1}
\]

General Aptitude

Q. No. 1 - 5 Carry One Mark Each

1. The ninth and the tenth of this month are Monday and Tuesday ___________.
   (A) figuratively  (B) retrospectively  (C) respectively  (D) rightfully

Key: (C)

2. 500 students are taking one or more courses out of Chemistry, Physics, and Mathematics. Registration records indicate course enrolment as follows: Chemistry (329). Physics (186).
Mathematics (295), Chemistry and Physics (83), Chemistry and Mathematics (217), and Physics and Mathematics (63). How many students are taking all 3 subjects?

(A) 37  (B) 43  (C) 47  (D) 53  

Key:  (D)  

Exp:  

Given  

\[ A + x_2 = 83 \] \,(1)  
\[ A + y_2 = 63 \] \,(2)  
\[ A + x_1 = 217 \] \,(3)  
And  

\[ x_1 + x_2 + A + x_3 = 329 \] \,(4)  
\[ x_2 + A + y_1 + y_2 = 186 \] \,(5)  
\[ x_3 + A + y_2 + z_1 = 295 \] \,(6)  
\[ x_1 + x_2 + x_3 + y_1 + y_2 + z_1 + A = 500 \] \,(7)  
\[ (1)+(2)+(3) \Rightarrow x_2 + y_2 + x_3 = 363 - 3A \] \,(8)  
\[ (4)+(5)+(6) \Rightarrow 3A + 2(363 - 3A) + (x_1 + y_1 + z_1) = 810 \]  
\[ \Rightarrow 3A + 2(363 - 3A) + (x_1 + y_1 + z_1) = 810 \] \,(\because \text{From (8)})  
\[ \Rightarrow -3A + 726 + (500 - x_2 - x_3 - y_1 - A) = 810 \] \,(\because \text{From (7)})  
\[ \Rightarrow -3A + 726 + (500 - 363 - 3A) - A = 810 \]  
\[ \Rightarrow 863 - A = 810 \Rightarrow A = 53 \]  
Alternate method  

\[ n(C) = 329, \quad n(P) = 186, \quad n(M) = 295, \quad n(C \cap P) = 83; \]  
\[ n(C \cup M) = 217, \quad (P \cap M) = 63 \]  
\[ n(P \cup C \cup M) = n(C) + n(P) + n(M) - n(C \cap P) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M). \]  
\[ \Rightarrow 500 = 329 + 186 + 295 - 83 - 217 - 63 + n(P \cap C \cap M) \]  
\[ \Rightarrow n(P \cap C \cap M) = 500 - 447 = 53. \]  

3.  It is _________ to read this year’s textbook __________ the last year’s.  

(A) easier, than  (B) most easy, than  (C) easier, from  (D) easiest, from  

Key:  (A)  

4.  Fatima starts from point P, goes North for 3 km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15 km in that direction. She then goes North for 6 km. How far is she from point P, and in which direction should she go to reach point P?  

(A) 8 km, East  (B) 12 km, North  (C) 6 km, East  (D) 10 km, North  

Key:  (A)
5. A rule states that in order to drink beer one must be over 18 years old. In a bar, there are 4 people. P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking beer. What must be checked to ensure that the rule is being followed?

(A) Only P’s drink  
(B) Only P’s drink and S’s age  
(C) Only S’s age  
(D) Only P’s drink, Q’s drink and S’s age

Key: (B)

Exp: For rules to be followed, we need to check P’s drink and S’s age.

Q. No. 6- 10 Carry Two Marks Each

6. Each of P, Q, R, S, W, X, Y and Z has been married at most once. X and Y have two children P and Q. Z is the grandfather of the daughter S of P. Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

(A) X is the mother-in-law of R  
(B) P and R are not married to each other  
(C) P is a son of X and Y  
(D) Q cannot be married to R

Key: (D)

7. The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

(A) 781  
(B) 791  
(C) 881  
(D) 891

Key: (C)

Exp: Total no. of 3 digit no’s = 9 \times 10 \times 10 = 900

The no. of 3-digit numbers in which ‘1’ is to the immediate right of 2 = 19

\[
\begin{array}{c}
2 \\
9 \text{ choices}
\end{array}
\xrightarrow{10 \text{ choices}}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
1 \\
2 \text{ \ (19 choices)}
\end{array}
\end{array}
\end{array}
\]

\[\therefore \text{ The no. of 3-digit no’s such that the digit 1 is never to immediate right of 2 is 900} – 19 = 881\]
Alternate method

Total no. of 3 digit numbers are \(= 9 \times 10 \times 10 = 900\).

Numbers with digit 1 is to the immediate right of 2 are

\[
\begin{align*}
2 & \, 1 & \, x \\
\times & \, 1 \times 10 & \, 9 \times 1 \times 1 \\
\hline \\
& 19 \\
\end{align*}
\]

So, no. of 3 digit numbers such that the digit 1 is never to the immediate right of 2 are \(= 900 - 19 = 881\)

8. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25m intervals in this plot.

Which of the following is the steepest path leaving from P?

(A) P to Q  
(B) P to R  
(C) P to S  
(D) P to T

Key:  (B)  

Exp: Closer lines represents steepest path  

Alternate method

The steepest path will be the path which is deepest from sea level. So, P to R is the steepest path.

9. 1200 men and 500 women can build a bridge in 2 weeks. 900men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

(A) 3000  
(B) 3300  
(C) 3600  
(D) 3900

Key:  (C)  

Exp: Given 1200 Men + 500 Women can build a bridge in 2 weeks. And 900 Men + 250 Women will take 3 weeks to build the same bridge.

\[\therefore\] To complete in a week; there are 2400 Men + 1000W required in the first equation and 2700 Men + 750 Women required in the second equation.

\[\therefore\] 2400 M + 1000W = 2700M + 750W

\[\Rightarrow 1W = \frac{6M}{5}\]
\[
\therefore \text{ The no. of men required to build the bridge in one week}
\]
\[
= 2400M + 1000\left(\frac{6M}{5}\right) = 3600 \text{ Men}
\]

Alternate method

Let a man can build the bridge in \( x \) weeks and a woman can build the bridge in \( y \) weeks.

\[
\frac{120}{x} + \frac{500}{y} = \frac{1}{2}
\]
\[
\frac{900}{x} + \frac{250}{y} = \frac{1}{3}
\]

By equations (i) and (ii); we get

\[
x = 3600; y = 3000
\]
\[
\Rightarrow \text{A man build the bridge 3600 weeks}
\]
\[
\Rightarrow \text{Required men 3600 to build in a week.}
\]

10. “If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective section, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters.”

Which of the following statements best reflects the author’s opinion?

(A) An intimate association does not allow for the necessary perspective.

(B) Matters are recorded with an impartial perspective.

(C) An intimate association offers an impartial perspective.

(D) Actors are typically associated with the impartial recording of matters.

Key: (A)