Mechanical Engineering
Q. No. 1 to 25 Carry One Mark Each

1. A mass m of a perfect gas at pressure $p_1$ and volume $V_1$ undergoes an isothermal process. The final pressure is $p_2$ and volume is $V_2$. The work done on the system is considered positive. If R is the gas constant and T is the temperature, then the work done in the process is

(A) $p_1 V_1 \ln \frac{V_2}{V_1}$
(B) $-p_1 V_1 \ln \frac{p_2}{p_1}$
(C) $RT \ln \frac{V_2}{V_1}$
(D) $-mRT \ln \frac{p_2}{p_1}$

Key: (B)

Exp: Isothermal work done, $W = -\int pdV$

For isothermal, $pV = C$

$p_1 V_1 = pV = C$ (constant)

so, $W = -p_1 V_1 \int \frac{dV}{V}$

$W = -p_1 V_1 \ln \left( \frac{V_2}{V_1} \right) = -p_1 V_1 \ln \left( \frac{p_2}{p_1} \right)$ ($\because p_1 V_1 = p_2 V_2$)

2. Which one of the following statements is TRUE for the ultrasonic machining (USM) process?
(A) In USM, the tool vibrates at subsonic frequency.
(B) USM does not employ magnetostrictive transducer.
(C) USM is an excellent process for machining ductile materials.
(D) USM often uses a slurry comprising abrasive-particles and water.

Key: (D)

3. The standard deviation of linear dimensions P and Q are 3 μm and 4 μm, respectively. When assembled, the standard deviation (in μm) of the resulting linear dimension (P+Q) is ________

Key: 5 to 5

Exp: Given that

Standard deviate of P is 3 μm $\Rightarrow$ Variance of P is 9 μm

Standard deviation of Q is 4 μm $\Rightarrow$ Variance of Q is 16 μm

Variance of P + Q = Var (P+Q) = Variance P + Variance Q = 9 + 16 = 25

$\therefore$ Standard deviation of P + Q = $\sqrt{\text{Variance}} = \sqrt{25} = 5$

4. The emissive power of a blackbody is $P$. If its absolute temperature is doubled, the emissive power becomes.

(A) 2P  (B) 4P  (C) 8P  (D) 16P

Key: (D)

Exp: Emissive power of black body $(E_0) = \sigma T^4$

Given $(E_0)_1 = P = \sigma T^4$
now $T \rightarrow 2T$

$(E_b)_2 = \sigma (2T)^4 = 16\sigma T^4 \Rightarrow (E_b)_2 = 16P$

5. The state of stress at a point is $\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$ and $\tau_{xy} = \tau_{yx} = 50$MPa.

The maximum normal stress (in MPa) at that point is __________

Key: 49.9 to 50.1

Exp: It is a pure torsion case in 2D

$\tau_{xy} = \tau_{yx} = 50$MPa

.: for pure Torsion,

$\tau_{xy} = \tau_{yx} = \sigma_i$

So maximum Normal stress $\sigma_i = 50$MPa

6. The determinant of a $2 \times 2$ matrix is 50. If one eigenvalue of the matrix is 10, the other eigenvalue is __________

Key: 5 to 5

Exp: Given that det of $2 \times 2$ Matrix is 50 and are Eigen Value is 10.

$\Rightarrow$ Other Eigen value is 5 (.: det = product of eigenvalues)

7. Which one of the following statement is TRUE?

(A) Both Pelton and Francis turbines are impulse turbines.

(B) Francis turbine is a reaction turbine but Kaplan turbine is an impulse turbine.

(C) Francis turbine is an axial – flow reaction turbine.

(D) Kaplan turbine is an axial – flow reaction turbine.

Key: (D)

8. Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is __________

Key: 0.75 to 0.75

Exp: Total No of outcomes when two coins are tossed is 4 and sample space

$S = \{ HH, HT, TH, TT \}$

Favorable out comes for existence of at least one head are HH, HT, TH.

Required probability $= \frac{3}{4} = 0.75$

9. A cantilever beam of length L and flexural modulus EI is subjected to a point load P at the free end. The elastic strain energy stored in the beam due to bending (neglecting transverse shear)

(A) $\frac{P^2L^4}{6EI}$

(B) $\frac{P^2L^4}{3EI}$

(C) $\frac{PL^4}{3EI}$

(D) $\frac{PL^4}{6EI}$

Key: (A)
10. It is desired to make a product having T-shaped cross-section from a rectangular aluminium block. Which one of the following processes is expected to provide the highest strength of the product?
(A) Welding (B) Casting (C) Metal Forming (D) Machining
Key: (C)

11. The heat loss from a fin is 6W. The effectiveness and efficiency of the fin are 3 and 0.75, respectively. The heat loss (in W) from the fin, keeping the entire fin surface at base temperature, is __________.
Key: 7.9 to 8.1

Exp: Given $Q_{loss} = 6W$
- $\varepsilon = 3$, $\eta = 0.75$
\[ \therefore \eta = \frac{Q_{act}}{Q_{max}} \]
- $Q_{max} = \frac{6}{0.75} = 8W$

12. For a single server with Poisson arrival and exponential service time, the arrival rate is 12 per hour. Which one of the following service rates will provide a steady state finite queue length?
(A) 6 per hour (B) 10 per hour (C) 12 per hour (D) 24 per hour
Key: (D)

Exp: $\lambda = 12 / hour$
For steady state finite queue length
- $\mu > \lambda$
So, $\mu = 24 / hour$

13. For the stability of a floating body the
(A) centre of buoyancy must coincide with the centre of gravity
(B) centre of buoyancy must be above the centre of gravity
(C) centre of gravity must be above the centre of buoyancy
(D) metacentre must be above the centre of gravity
Key: (D)

Exp: Stability of floating body is measure with the help of Meta center.
Floating body to be stable, Meta center must be above C.G
\[ \therefore MG = \frac{I}{V} - BG \]
For stability $MG > 0$
14. The divergence of the vector \(-yi + xj\) \[__________\]

**Key:** 0 to 0

**Exp:** Let \(\vec{F} = -yi + xj\)

\[
\text{divergence of } \vec{F} = \frac{\partial}{\partial x} (-y) + \frac{\partial}{\partial y} (x) = 0
\]

15. For a loaded cantilever beam of uniform cross-section, the bending moment (in N.mm) along the length is \(M(x) = 5x^2 + 10x\), where \(x\) is the distance (in mm) measured from the free end of the beam. The magnitude of shear force (in N) in the cross-section at \(x = 10\) mm is \[__________\].

**Key:** 110 to 110

**Exp:**

\[
\therefore S.F = \frac{d}{dx} (B.M)
\]

\[
S.F = \frac{d}{dx} \left( 5x^2 + 10x \right)
\]

\[
S.F = 10x + 10 \Rightarrow (S.F)_{x=10} = (10 \times 10) + 10 = 110N
\]

16. A sample of 15 data is as follows: 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is (A) 4 (B) 13 (C) 17 (D) 20

**Key:** (C)

**Exp:** We know that mode is the value of the data which occurred most of the time. Hence, 17 is the mode.

17. If a mass of moist air contained in a closed metallic vessel is heated, then its relative humidity decreases (A) relative humidity decreases (B) relative humidity increases (C) specific humidity increases (D) specific humidity decreases

**Key:** (A)

**Exp:** Given that mass of moist air contained in a closed metallic vessel is heated, means its specific humidity (\(\omega\)) is constant. So, from the psychrometric chart,

\[
\text{Relative humidity} \quad (\phi) = 100\%
\]

(DBT)

\[
\phi_1 \quad 1 \quad 2 \quad \phi_2
\]

We can say that at constant specific humidity, as temperature increases relative humidity decreases i.e., \(\phi_2 < \phi_1\)

So, Answer is (A)
18. In a slider-crank mechanism, the lengths of the crank and the connecting rod are 100mm and 160mm, respectively. The crank is rotating with an angular velocity of 10 radian/s counterclockwise. The magnitude of linear velocity (in m/s) of the piston at the instant corresponding to the configuration shown in the figure is ________________

Key: 0.99 to 1.01

Exp: 
\[ V = r \omega \sin \theta + \frac{20}{n} \]

if \( \theta = 90 \)

\[ V = r \omega \sin 90^\circ + \frac{20}{2} = 0.1 \times 1(\sin 90^\circ) = 1 \text{ m/sec} \]

19. A machine component made of a ductile material is subjected to a variable loading with \( \sigma_{\text{min}} = -50 \text{ MPa} \) and \( \sigma_{\text{max}} = 50 \text{ MPa} \). If the corrected endurance limit and the yield strength for the material are \( \sigma'_e = 100 \text{ MPa} \) and \( \sigma_y = 300 \text{ MPa} \), the factor of safety is __________

Key: 1.99 to 2.01

Exp:
\[ \sigma_{\max} = 50 \text{ MPa} \]
\[ \sigma_{\min} = -50 \text{ MPa} \]
\[ \sigma_y = 300 \text{ MPa} \]
\[ \sigma_e = 100 \text{ MPa} \]

\[ \sigma_{\max} = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{50 - 50}{2} = 0 \]
\[ \sigma_y = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{50 - (-50)}{2} = 50 \]

\[ \frac{\sigma_{\max} + \sigma_e}{\sigma_{\text{st}}} = \frac{1}{\text{FS}} \]
\[ \frac{0 + 50}{300} = \frac{1}{\text{FS}} \]

FS = 2

20. The crystal structure of aluminium is

(A) body-centred cubic
(B) face-centred cubic
(C) close-packed hexagonal
(D) body-centred tetragonal

Key: (B)

21. A steel bar is held by two fixed supports as shown in the figure and is subjected to an increase of temperature \( \Delta T = 100^\circ C \) and 200GPa, respectively, the magnitude of thermal stress (in MPa) induced in the bar is __________
22. The Laplace transform of \(te^t\) is

\[
(A) \frac{s}{(s+1)^2} \quad (B) \frac{1}{(s-1)^2} \quad (C) \frac{1}{(s+1)^2} \quad (D) \frac{s}{s-1}
\]

Key: (B)

Exp: 
\[
L\{te^t\} = \frac{1}{(s-1)^2}; \quad \because L\{e^{at}f(t)\} = F(s-a)
\]
where \(F(s)=L\{f(t)\}\)

23. Consider a laminar flow at zero incidence over a flat plate. The shear stress at the wall is denoted by \(\tau_w\). The axial positions \(x_1\) and \(x_2\) on the plate are measured from the leading edge in the direction of flow. If \(x_2 > x_1\), then

(A) \(\tau_w\bigg|_{x_1} = \tau_w\bigg|_{x_2} = 0\)  \quad (B) \(\tau_w\bigg|_{x_1} = \tau_w\bigg|_{x_2} \neq 0\)  \quad (C) \(\tau_w\bigg|_{x_1} > \tau_w\bigg|_{x_2}\)  \quad (D) \(\tau_w\bigg|_{x_1} < \tau_w\bigg|_{x_2}\)

Key: (C)

Exp: 
\[
\tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \bigg|_{x_1} > \tau_w = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \bigg|_{x_2}
\]

24. A mass \(m\) is attached to two identical springs having spring constant \(k\) as shown in the figure. The natural frequency \(\omega\) of this single degree of freedom system is

(A) \(\sqrt{\frac{2k}{m}}\)  \quad (B) \(\sqrt{\frac{k}{m}}\)  \quad (C) \(\sqrt{\frac{k}{2m}}\)  \quad (D) \(\sqrt{\frac{4k}{m}}\)

Key: (A)

Exp: Equivalent stiffness \(k_{eq} = k + k = 2k\)

\[
\text{Natural frequency, } (\omega_n) = \sqrt{\frac{k_{eq}}{m}} \Rightarrow \omega_n = \sqrt{\frac{2k}{m}}
\]

25. Given the atomic weight of Fe is 56 and that of C is 12, the weight percentage of carbon in cementite (Fe\(_3\)C) is 6.3 to 7.0.

Key: 6.3 to 7.0

Exp: Percentage of carbon by weight in cementite = \(\frac{12}{56 \times 3 + 12} \times 100 = 6.67\%\)
Q. No. 26 to 55 Carry Two Marks Each

26. In an orthogonal machining with a tool of 9° orthogonal rake angle, the uncut chip thickness is 0.2mm. The chip thickness fluctuates between 0.25 mm and 0.4 mm. The ratio of the maximum shear angle to the minimum shear angle during machining is __________

Key: 1.45 to 1.53

Exp:
\[ \alpha = 9^\circ \]
\[ t_i = 0.2\text{mm} \]
\[ t_c = 0.25\text{mm to } 0.4\text{mm} \]
\[ \therefore \tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} \text{, where } r = \frac{t_i}{t_c} \]
if \( t_c = 0.25\text{mm} \), \( r = 0.8 \) & \( \phi = 42.08^\circ \)
if \( t_c = 0.4\text{mm} \), \( r = 0.5 \) & \( \phi = 28.18^\circ \)
\[ \frac{\phi_{\text{max}}}{\phi_{\text{min}}} = \frac{42.08}{28.18} = 1.493 \]

27. A cylindrical pin of 25.0025 ± 0.005 mm diameter is electroplated. Plating thickness is 2.005 ± 0.002 mm. Neglecting the gauge tolerance, the diameter (in mm, up to 3 decimal points accuracy) of the GO ring gauge to inspect the plated pin is ________.

Key: 29.030 to 29.030

Exp: Diameter of GO end of ring gauge = 25.02 + 2(2.005) = 29.030mm

28. A helical compression spring made of wire of circular cross-section is subjected to a compressive load. The maximum shear stress induced in the cross-section of the wire is 24 MPa. For the same compressive load, if both the wire diameter and the mean coil diameter are doubled, the maximum shear stress (in MPa) induced in the cross-section of the wire is ______.

Key: 6 to 6

Exp:
\[ \tau_{\text{max}} = \frac{8P.D}{\pi d^3} \]
Given, \( \tau_1 = 24\text{MPa} \)
\( P_1 = P_2, d_2 = 2d_1, D_2 = 2D_1 \)
so, \( \frac{\tau_1}{\tau_2} = \frac{D_1}{D_2} \cdot \frac{d_1^3}{d_2^3} \Rightarrow \frac{24}{\tau_2} = \frac{1}{2} \)
\[ \tau_2 = \frac{48}{2} = 24\text{MPa} \]
\[ \therefore \tau_2 = 6\text{MPa} \]

29. In a counter-flow heat exchanger, water is heated at the rate of 1.5kg/s from 40°C to 80°C by an oil entering at 120°C and leaving at 60°C. The specific heats of water and oil are 4.2kJ/kg.K and 2kJ/kg.K respectively. The overall heat transfer coefficient is 400 W/m².K. The required heat transfer surface area (in m²) is

(A) 0.104 (B) 0.022 (C) 10.4 (D) 21.84

Key: (D)

Exp: Given counter flow Heat Exchanger
Water - \( m_w = 1.5 \text{ kg/s} \)  
\( T_{i_w} = 40^\circ \text{C} \)  
\( T_{e_w} = 80^\circ \text{C} \)  
\( C_{p_w} = 4.2 \text{ kJ/kg.K} \)

Oil - \( T_{h_o} = 120^\circ \text{C} \)  
\( T_{i_o} = 40^\circ \text{C} \)  
\( T_{e_o} = 60^\circ \text{C} \)  
\( C_{p_o} = 2 \text{ kJ/kg.K} \)

Overall Heat Transfer Coefficient = 400 W/m\(^2\)K
Area (A) = \( \frac{Q}{UA(LMTD)} \)

\[ \frac{m_w \cdot C_{p_w} \cdot (T_{e_w} - T_{i_w})}{U \left( \frac{\theta_i - \theta_e}{\ln(\theta_e / \theta_i)} \right)} = \frac{1.5 \times 4.2 \times (80 - 40) \times 10^3}{400 \left[ \frac{40 - 20}{\ln(40/20)} \right]} = 21.83 \text{ m}^2 \Rightarrow A = 21.83 \text{ m}^2 \]

30. The rod PQ of length \( L = \sqrt{2} \text{ m} \) and uniformly distributed mass of \( M = 10 \text{ kg} \), is released from rest at the position shown in the figure. The ends slide along the frictionless faces OP and OQ. Assume acceleration due to gravity, \( g = 10 \text{ m/s}^2 \). The mass moment of inertia of the rod about its centre of mass and an axis perpendicular to the plane of the figure is \( (ML^2)/12 \). At this instant, the magnitude of angular acceleration (in radian/s\(^2\)) of the rod is \___________\.
31. A steel plate, connected to a fixed channel using three identical bolts A, B and C, carries a load of 6kN as shown in the figure. Considering the effect of direct load and moment, the magnitude of resultant shear force (in kN) on bolt C is.

\[ M = \left( \frac{P \times e}{r_1^2 + r_2^2} \right) \times \frac{1500}{(50^2 + 50^2)} = 0.3 \]

\[ R_A^1 = R_C^1 = C \times r_1 = 0.3 \times 50 = 15 \text{kN} \]

Resultant shear at C = 15 + 2 = 17 kN.

Key: (C)

Exp: Primary shear \( \frac{6}{3} = 2 \text{kN} \)

Secondary shear 1500 kN – mm

\[ PV = mRT \]

\[ \frac{1.5 \times 10^5 \times 2.87}{287 \times 300} = 5 \text{kg} \]

32. The volume and temperature of air (assumed to be an ideal gas) in a closed vessel is 2.87 m\(^3\) and 300K, respectively. The gauge pressure indicated by a manometer fitted to the wall of the vessel is 0.5 bar. If the gas constant of air is \( R = 287 \text{ J/kg.K} \) and the atmospheric pressure is 1 bar, the mass of air (in kg) in the vessel is

Key: (C)

Exp: \[ V = 2.87 \text{ m}^3; \quad T = 300 \text{K} \]

\[ \rho_{\text{gauge}} = 0.5 \text{bar} \]

\[ R = 287 \text{ J/kg.K} \]

\[ \rho_{\text{atm}} = 1 \text{bar} \]

\[ \rho_{\text{abs}} = \rho_{\text{g}} + \rho_{\text{atm}} = 1.5 \text{bar} \]

\[ PV = mRT \]

\[ \frac{1.5 \times 10^5 \times 2.87}{287 \times 300} = 5 \text{kg} \]

33. For the laminar flow of water over a sphere, the drag coefficient \( C_F \) is defined as \( C_F = \frac{F}{(\rho U^2 D^2)} \), where \( F \) is the drag force, \( \rho \) is the fluid density, \( U \) is the fluid velocity and \( D \) is the diameter of the sphere. The density of water is 1000 kg/m\(^3\). When the diameter of the sphere is 100 mm and the fluid velocity is 2 m/s, the drag coefficient is 0.5. If water now flows over another sphere of diameter 200 mm under dynamically similar conditions, the drag force (in N) on this sphere is _____________.

Key: (C)

Exp: Primary shear \( \frac{6}{3} = 2 \text{kN} \)

Secondary shear 1500 kN – mm

\[ PV = mRT \]

\[ \frac{1.5 \times 10^5 \times 2.87}{287 \times 300} = 5 \text{kg} \]
Key: 19.9 to 20.1

Exp: Given that the condition is dynamic similarity, and in the given condition, Inertia and viscous force plays major role, hence Reynold’s number should be same for both model and prototype.

\( (Re)_1 = (Re)_2 \)

In the first case: \( U_1 = 2 \text{ m/sec}, D_1 = 100 \text{ mm}, \rho = 1000 \text{ kg/m}^3 \)

In the second case: \( U_2 = 2 \text{ m/sec}, D_2 = 200 \text{ mm}, \rho = 1000 \text{ kg/m}^3 \)

\[ \left( \frac{\rho U D}{\mu} \right)_1 = \left( \frac{\rho U D}{\mu} \right)_2 \]

Since same water is flowing over both sphere

\( \mu_1 = \mu_2, \rho_1 = \rho_2 \)

\( U_1 D_1 = U_2 D_2 \)

\( \Rightarrow (2)(100) = (V_2)(200) \)

\( U_2 = 1 \text{ m/sec} \)

So, Drage force in second case will be

\[ F_2 = C \rho U^2 D^2 = (0.5)(1000)(1)^2(0.2)^2 = 20 \text{ N} \]

34. A rod of length 20mm is stretched to make a rod of length 40 mm. Subsequently, it is compressed to make a rod of final length 10mm. Consider the longitudinal tensile strain as positive and compressive strain as negative. The total true longitudinal strain in the rod is

(A) –0.5 
(B) –0.69 
(C) –0.75 
(D) –1.0

Key: (B)

Exp: \( \ell_i = 20 \text{ mm} \quad \ell_f = 10 \text{ mm} \)

\( \therefore \) true strain,

\[ \varepsilon_{\text{true}} = \varepsilon_n \left( \frac{\ell_f}{\ell_i} \right) = \varepsilon_n \left( \frac{10}{20} \right) = -0.69 \]

35. A gear train shown in the figure consists of gears P, Q, R and S. Gear Q and gear R are mounted on the same shaft. All the gears are mounted on parallel shafts and the number of teeth of P, Q, R and S are 24, 45, 30 and 80, respectively. Gear P is rotating at 400 rpm. The speed (in rpm) of the gear S is _________.

Key: 120 to 120

Exp: \( T_p = 24, \quad T_Q = 45, \quad T_R = 30, \quad T_S = 80 \)

\( N_p = 400 \text{ rpm} \)

\( N_p . T_p = N_Q . T_Q \)
36. In the Rankine cycle for a steam power plant the turbine entry and exit enthalpies are 2803 kJ/kg and 1800 kJ/kg, respectively. The enthalpies of water at pump entry and exit are 121 kJ/kg and 124 kJ/kg, respectively. The specific steam consumption (in kg/kW.h) of the cycle is _____

Key: 3.5 to 3.7

Exp: Given that,

\[ h_1 = 2803 \text{ kJ/kg}, \quad h_2 = 1800 \text{kJ/kg} \]
\[ h_3 = 121 \text{kJ/kg}, \quad h_4 = 124 \text{kJ/kg} \]
\[ W_t = h_1 - h_2 \]
\[ = 2803 - 1800 = 1003 \text{kJ/kg} \]
\[ W_p = h_4 - h_3 \]
\[ = 124 - 121 = 3 \text{kJ/kg} \]
\[ W_{net} = W_t - W_p = 1000 \text{kJ/kg} \]

Specific Steam consumption \( \frac{3600}{W_{net}} = 3.6 \text{kg/kWh} \)

37. A calorically perfect gas (specific heat at constant pressure 1000 J/kg.K) enters and leaves a gas turbine with the same velocity. The temperatures of the gas at turbine entry and exit are 1100 K and 400 K, respectively. The power produced is 4.6 MW and heat escapes at the rate of 300 kJ/s through the turbine casing. The mass flow rate of the gas (in kg/s) through the turbine is.

(A) 6.14  (B) 7.00  (C) 7.50  (D) 8.00

Key: (B)

Exp: Given that,

\[ C_p = 1000 \text{J/kg.K} \]
\[ T_1 = 1100 \text{K}, \quad P = 4.6 \text{MW} \]
\[ T_2 = 400 \text{K}, \quad Q_L = 300 \text{kJ/s} \]
\[ E_{in} = E_{out} \]
\[ h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2 + Q_L + P \]
\[ \therefore V_1 = V_2, \quad Z_1 = Z_2 \]
\[ \text{So, } h_1 - h_2 = Q_L + P \]
\[ m C_p (T_1 - T_2) = Q_L + P \]
\[ \therefore m = \frac{(300 \times 10^3) + (4.6 \times 10^6)}{1000 \times (1100 - 400)} \]
\[ \therefore m = 7 \text{kg/sec} \]
38. Three masses are connected to a rotating shaft supported on bearings A and B as shown in the figure. The system is in a space where the gravitational effect is absent. Neglect the mass of shaft and rods connecting the masses. For \( m_1 = 10 \text{kg}, \ m_2 = 5 \text{kg} \) and \( m_3 = 2.5 \text{kg} \) and for a shaft angular speed of 1000 radian/s, the magnitude of the bearing reaction (in N) at location B is _________.

Key: 0 to 0

Exp: \( m_1 = 10 \text{kg}, \ r_1 = 0.1 \text{m} \)
\( m_2 = 5 \text{kg}, \ r_2 = 0.2 \text{m} \)
\( m_3 = 2.5 \text{kg}, r_3 = 0.4 \text{m} \)

\[
\sum F_x = m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + m_3 r_3 \cos \theta_3 \\
= (10 \times 0.1) \cos 0 + (5 \times 0.2) \cos 120 + (2.5 \times 0.4) \cos 240 \\
= 1 - 0.5 - 0.5 = 0
\]

\[
\sum F_y = m_1 r_1 \sin \theta_1 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3 \\
= 0 + (5 \times 0.2) \sin 120 + (2.5 \times 0.4) \sin 240 = 0
\]

\( R_A = R_B = 0 \text{N} \)

39. A strip of 120 mm width and 8mm thickness is rolled between two 300 mm-diameter rolls to get a strip of 120 mm width and 7.2 mm thickness. The speed of the strip at the exit is 30 m/min. There is no front or back tension. Assuming uniform roll pressure of 200 MPa in the roll bite and 100% mechanical efficiency, the minimum total power (in kW) required to drive the two rolls is _________.

Key: 9.4 to 9.8

Exp: Width = 120mm

Initial thickness to = 8mm

Diameter of Roller = 300mm

Radius of Roller = 150mm

Final thickness = 7.2mm

\( \Delta h = t_i - t_f = 8 - 7.2 = 0.8 \text{mm} \)

Power require to drive one roller

\[
P = T.\omega = F \times L_p \times \omega = \sigma A L_p \omega \quad \text{Here, } A = L_p \times b
\]

\[
P = \sigma L_p^2 b \omega \quad \text{here, } L_p = \sqrt{R\Delta h}
\]

\[
P = \sigma (R\Delta h) b \omega \left( \frac{V}{R} \right)
\]

\[
= 200 \times 10^6 \times 0.8 \times 10^{-3} \times 0.12 \times \frac{30}{60} = 4.8 \text{ kW}
\]

So, power require to drive 2 roller = 2P = 2 \times 4.8 kW = 9.6 kW
40. A product made in two factories P and Q, is transported to two destinations, R and S. The per unit costs of transportation (in Rupees) from factories to destinations are as per the following matrix:

<table>
<thead>
<tr>
<th>Factory</th>
<th>Destination</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td></td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Factory P produces 7 units and factory Q produces 9 units of the product. Each destination requires 8 units. If the north-west corner method provides the total transportation cost as X (in Rupees) and the optimized (the minimum) total transportation cost Y (in Rupees), then (X-Y), in Rupees, is

(A) 0  (B) 15  (C) 35  (D) 105

Key: Answer is not matched with IIT Key

Exp:

By North West corner Rule

Total cost, \( x = 10 \times 7 + 7 \times 1 + 4 \times 8 = \text{Rs.105} \)

By VAM

\( u_1 = 0 \)  \( V_1 = 6 \)  \( V_2 = 7 \)

\( u_2 = -3 \)  \( 7 \)  \( (1) \)

\( 8 \)  \( 8 \)

Total minimum Cost, \( y = 3 \times 8 + 4 \times 1 + 7 \times 7 = \text{Rs.77} \)

So, \( x-y = \text{Rs.28} \)

41. One kg of an ideal gas (gas constant \( R = 287 \text{ J/kg.K} \)) undergoes an irreversible process from state-1 (1 bar, 300 K) to state-2 (2 bar, 300 K). The change in specific entropy \( (s_2 - s_1) \) of the gas (in J/kg. K) in the process is __________

Key: \(-201\) to \(-197\)

Exp:

Given \( m = 1 \text{ kg}, \) \( R = 287 \text{ J/kg.K} \)

\( P_1 = 1 \text{ bar}, \) \( P_2 = 2 \text{ bar} \)

\( T_1 = 300 \text{ K}, \) \( T_2 = 300 \text{ K} \)

\( \because \) Same Temperature

\( S_2 - S_1 = -R \ln \left( \frac{P_2}{P_1} \right) = -287 \ln \left( \frac{2}{1} \right) = -198.93 \text{ J/kg K} \)
42. A 60 mm-diameter water jet strikes a plate containing a hole of 40 mm diameter as shown in the figure. Part of the jet passes through the hole horizontally, and the remaining is deflected vertically. The density of water is 1000 kg/m$^3$. If velocities are as indicated in the figure, the magnitude of horizontal force (in N) required to hold the plate is _________

**Key:** 627 to 629

**Exp:**

Force in X–direction = Rate of change of momentum

\[
= (P_1 - P_r)_{\text{X-direction}}
\]

\[
= \left( m_1 V_1 - m_2 V_2 \right)_{\text{X-direction}}
\]

\[
= V \left( \rho A_1 V_1 - \rho A_2 V_2 \right) \quad (\because V_1 = V_2)
\]

\[
= \rho V^2 \left( A_1 - A_2 \right)
\]

\[
= 1000 \times 20 \times 20 \times \frac{\pi}{4} \left[ (0.06)^2 - (0.04)^2 \right]
\]

\[
= 628.32 \text{ N}
\]

43. The arrangement shown in the figure measures the velocity $V$ of a gas of density 1 kg/m$^3$ flowing through a pipe. The acceleration due to gravity is 9.81 m/s$^2$. If the manometric fluid is water (density 1000 kg/m$^3$) and the velocity $V$ is 20 m/s, the differential head $h$ (in mm) between the two arms of the manometer is __________

**Key:** 19 to 21

**Exp:**

Given

\[
\rho_g = 1 \text{ kg/m}^3, \quad g = 9.81 \text{ m/s}^2
\]

\[
\rho_m = 1000 \text{ kg/m}^3, \quad V = 20 \text{ m/s}
\]

\[
V = \sqrt{2gH} \quad \text{Where} \quad H = h \left( \frac{\rho_m}{\rho} - 1 \right)
\]

\[
\frac{V^2}{2g} = h \left( \frac{\rho_m}{\rho} - 1 \right)
\]

\[
\left[ \frac{(20)^2}{2 \times 9.81} \right] = h(1000 - 1)
\]

\[
h = 0.0204m = 2.04 \text{ cm} \quad \therefore h = 20.4 \text{ mm}
\]
44. A metal ball of diameter 60mm is initially at 220 °C. The ball is suddenly cooled by an air jet of 20°C. The heat transfer coefficient is 200 W/m².K and 9000kg/m³, respectively. The ball temperature (in °C) after 90 seconds will be approximately.

(A) 141 (B) 163 (C) 189 (D) 210

Key: (A)

Exp:
\[ D = 60\text{mm} = 0.06\text{m} \]
\[ h = 200\text{W/m}^2\text{K}, \quad \rho = 9000\text{kg/m}^3 \]
\[ T_i = 220\text{°C}, \quad C_p = 400\text{J/kgK}, \quad K = 400\text{W/mK} \]
\[ t = 90\text{sec} \quad T = ? \]
\[ T - T_e = e^{\frac{h.A}{\rho V C}} \quad \Rightarrow \quad \frac{220 - 20}{T_i - 20} = e^{\frac{200\times90+3}{0.03\times9000\times400}} \]
\[ T_f = 141.3\text{°C} \]

45. A single – plate clutch has a friction disc with inner and outer radii of 20mm and 40mm, respectively. The friction lining in the disc is made in such a way that the coefficient of friction \( \mu \) varies radially as \( \mu = 0.01r \), where \( r \) is in mm. The clutch needs to transmit a friction torque of 18.85kN.mm. As per uniform pressure theory, the pressure (in MPa) on the disc is

Key: 0.49 to 0.51

Exp:
\[ r_i = 20\text{mm}, \quad r_o = 40\text{mm}, \quad \mu = 0.01r \]
\[ T = 18.85\text{kN}\cdot\text{mm} \]
\[ p = ? \]
\[ P = \int p 2\pi rd = \int (0.01r) 2\pi r dr = 0.0628 r^3 \text{dr} \]
\[ T = 0.0628 p \int _{20}^{40} r^3 \text{dr} = 0.0628 p \left[ \frac{r^4}{4} \right] _{20}^{40} \]
\[ 18.85 \times 10^3 = 0.0628 p \left[ \frac{40^4}{4} - \frac{20^4}{4} \right] \]
\[ p = 0.5\text{MPa} \]

46. The surface integral \( \iint _S \vec{F}.n\text{d}S \) over the surface \( S \) of the sphere \( x^2 + y^2 + z^2 = 9 \), where \( \vec{F} = (x+y)i + (x+z)j + (y+z)k \) and \( n \) is the unit outward surface normal, yields ______.

Key: 225 to 227

Exp:
\[ \vec{F} = (x+y)i + (x+z)j + (y+z)k \]
\[ \text{div}\vec{F} = \frac{\partial}{\partial x}(x+y) + \frac{\partial}{\partial y}(x+z) + \frac{\partial}{\partial z}(y+z) = 1 + 0 + 1 = 2 \]

By divergence theorem,
\[ \iint _S \vec{F} \cdot n\text{d}S = \iiint _V \text{div}\vec{F}\text{d}V \]
\[ \text{where } V \text{ is volume of given surface of sphere } x^2 + y^2 + z^2 = 9 \]
\[ = \iiint _V 2\text{d}V = 2 \times 4\pi \left( \frac{27}{3} \right) = 72\pi = 226.1947 \]
47. Block 2 slides outward on link 1 at a uniform velocity of 6 m/s as shown in the figure. Link 1 is rotating at a constant angular velocity of 20 radian/s counterclockwise. The magnitude of the total acceleration (in m/s²) of point P of the block with respect to fixed point O is

Key: 243 to 244

Exp: Acceleration of the block,

\[ v = 6 \text{ m/s}, \ \omega = 20 \text{ rad/sec} \]

\[ a = \sqrt{a_r^2 + a_v^2} = \sqrt{\left(\omega^2 r\right)^2 + (2v\omega)^2} \]

\[ r = OP, \]

\[ a_r = 2v\omega = 2 \times 6 \times 20 = 240 \text{ m/s}^2 \]

\[ a_v = \omega^2 r = (20)^2 \times 0.1 = 40 \]

\[ a = \sqrt{240^2 + 40^2} = 243.31 \text{ m/s}^2 \]

48. During the turning of a 20mm-diameter steel bar at a spindle speed of 400 rpm, a tool life of 20 minute is obtained. When the same bar is turned at 200 rpm, the tool life becomes 60 minute. Assume that Taylor’s tool life equation is valid. When the bar is turned at 300 rpm, the tool life (in minute) is approximately.

(A) 25  (B) 32  (C) 40  (D) 50

Key: (B)

Exp: Taylor’s Tool life equation,

\[ VT^n = C \]

\[ \therefore V_1T_1^n = V_2T_2^n \]

\[ \left(\frac{T_2}{T_1}\right)^n = \frac{V_1}{V_2} \]

\[ \left(\frac{60}{20}\right)^n = \frac{400}{200} \]

\[ n = 0.63 \]

at \( N_3 = 300 \text{ rpm} \), \( T_3 = ? \)

\[ \left(\frac{T_3}{T_1}\right)^n = \frac{V_1}{V_3} \]

\[ \left(\frac{T_3}{20}\right)^{0.63} = \frac{400}{300} \]

\[ T_3 = 31.57 \text{ min utes} \]

\[ T_3 = 32 \text{ min utes} \]
49. Consider the matrix \( A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix} \) whose eigenvectors corresponding to eigenvalues \( \lambda_1 \) and \( \lambda_2 \) are \( x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix} \) and \( x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix} \), respectively. The value of \( x_1^T x_2 \) is __________.

Key: 0 to 0

Exp: 
\( A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix} \)

Eigen vectors are \( X_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix} \); \( X_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix} \)

Where \( \lambda_1, \lambda_2 \) Eigen values of \( A \)

\[
X_1^T X_2 = 70(\lambda_2 - 80) + (\lambda_1 - 50)70
\]

\[
= 70\lambda_2 - 5600 + 70\lambda_1 - 3500 = 70(\lambda_1 + \lambda_2) - 9100
\]

\[
= 70(130) - 9100 = 9100 - 9100 = 0
\]

\( \therefore \) sum of eigen values = \( \lambda_1 + \lambda_2 \)

Trace = 50 + 80 = 130

50. The radius of gyration of a compound pendulum about the point of suspension is 100mm. The distance between the point of suspension and the centre of mass is 250mm. Considering the acceleration due to gravity as 9.81 m/s\(^2\), the natural frequency (in radian/s) of the compound pendulum is __________.

Key: 15 to 16

Exp: 
\( k = 100 \text{mm} = 0.1 \text{m} \)

\( L = 250 \text{mm} = 0.250 \text{m} \)

\( g = 9.81 \text{m/s}^2 \)

\( I = mk^2 = m(0.1)^2 \)

\( W_n = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{m \times 9.81 \times 0.250}{m(0.1)}} = 15.66 \text{rad/sec} \)

51. Consider the differential equation \( 3y''(x) + 27y(x) = 0 \) with initial conditions \( y(0) = 0 \) and \( y'(0) = 2000 \). The value of \( y \) at \( x = 1 \) is __________.

Key: 93 to 95

Exp: 
\( 3y''(x) + 27y(x) = 0, y(0) = 0, y'(0) = 2000 \)

Auxiliary equation, \( 3m^2 + 27 = 0 \Rightarrow m^2 + 9 = 0 \Rightarrow m = 0 + 3i \)

\( y_e = c_1 \cos 3x + c_2 \sin 3x \)

\( y(0) = 0 \Rightarrow c_1 + 0 = 0 \Rightarrow c_1 = 0 \)

\( y_e = c_2 \sin 3x \)

\( y' = 3c_2 \cos 3x \)
52. If \( f(z) = (x^2 + ay^2) + ibxy \) is a complex analytic function of \( z = x + iy \), where \( i = \sqrt{-1} \), then
   (A) \( a = -1, b = -1 \)  
   (B) \( a = -1, b = 2 \)  
   (C) \( a = 1, b = 2 \)  
   (D) \( a = 2, b = 2 \)

   Key:  (B)

   Exp: Given \( f(z) = (x^2 + ay^2) + ibxy \) is analytic
   \[ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial x}{\partial y} = -\frac{\partial v}{\partial x} \]
   \[ \text{value } u = x^2 + ay^2, \quad v = bxy \]
   \[ \frac{\partial u}{\partial x} = 2x, \quad \frac{\partial v}{\partial x} = by \]
   \[ \frac{\partial u}{\partial y} = 2ay, \quad \frac{\partial v}{\partial y} = bx \]

   Clearly for \( b = 2 \) and \( a = -1 \) above Cauchy-Riemann equations holds

53. A project starts with activity A and ends with activity F. The precedence relation and durations of the activities are as per the following table:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Immediate Predecessor</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>7</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>14</td>
</tr>
<tr>
<td>E</td>
<td>C</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>D,E</td>
<td>9</td>
</tr>
</tbody>
</table>

   The minimum project completion time (in days) is ______

   Key:  30 to 30

   Exp:

   \[ \therefore \text{Minimum project completion Time} = \text{Length of longest path} = 4+3+14+9=30 \text{ Days.} \]

54. Maximize \( Z = 5x_1 + 3x_2 \)

   Subject to
   \[ x_1 + 2x_2 \leq 10, \]
   \[ x_1 - x_2 \leq 8, \]
   \[ x_1, x_2 \geq 0. \]

   In the starting Simplex tableau, \( x_1 \) and \( x_2 \) are non-basic variables and the value of \( Z \) is zero. The value of \( Z \) in the next Simplex tableau is __________.
55. The principal stresses at a point in a critical section of a machine component are \( \sigma_1 = 60 \text{MPa}, \sigma_2 = 5 \text{MPa} \) and \( \sigma_3 = -40 \text{MPa} \). For the material of the component, the tensile yield strength is \( \sigma_y = 200 \text{MPa} \). According to the maximum shear stress theory, the factor of safety is

(A) 1.67  
(B) 2.00  
(C) 3.60  
(D) 4.00

Key: (B)

Exp:
\[
\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_y}{2} \Rightarrow \sigma_1 - \sigma_3 = \frac{\sigma_y}{\text{FOS}}
\]
\[
60 - (-40) = \frac{200}{\text{FOS}}
\]
\[
\text{FOS} = 2
\]

Alternate Method:
Let \( a - b = 2K \) \( \Rightarrow a = b + 2K \)
\[
ab - b = (b + 2K)b - b = b^2 + 2Kb - b = (b^2 - b) + 2Kb = b(b - 1) + 2Kb
\]
Even; since 2Kb always even & b (b-1) also even, if with b is odd or even.
4. A couple has 2 children. The probability that both children are boys if the older one is a boy is
   (A) 1/4  (B) 1/3  (C) 1/2  (D) 1

   **Key:**  (C)

   **Exp:** Probability = \( \frac{\text{No. of Favourable cases}}{\text{Total No. of Possible cases}} \)

   \( \therefore \) The probability that both children are boys if the older one is a boy = \( \frac{1}{2} \)

5. The ways in which this game can be played _________ potentially infinite.
   (A) is  (B) is being  (C) are  (D) are being

   **Key:**  (C)

   **Q. No. 6-10 Carry Two Marks Each**

6. “If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters.”

   Which of the following closest in meaning to “cleaving”?
   (A) Deteriorating  (B) Arguing  (C) Departing  (D) Splitting

   **Key:**  (D)

7. There are 4 women P, Q, R, S, and 5 men V, W, X, Y, Z in a group. We are required to form pairs each consisting of one woman and one man. P is not to be paired with Z, and Y must necessarily be paired with someone. In how many ways can 4 such pairs be formed?
   (A) 74  (B) 76  (C) 78  (D) 80

   **Key:**  (C)

   **Exp:**
   If P is paired with y; they
   Q has 4 choices
   R has 3 choices
   S has 2 choices
   Total 24 choices

   (or)

   If Q is paired with y; then
   P has 3 choices
   R has 3 choices
   S has 2 choices
   Total 18 choices

   (or)

   If R is paired with y; then
   P has 3 choices
   Q has 3 choices
   S has 2 choices
   Total 18 choices
If S is paired with y; then

P has 3 choices
Q has 3 choices
S has 2 choices

Total 18 choices

∴ Total number of ways = 24+18+18+18=78

8. In the graph below, the concentration of a particular pollutant in a lake is plotted over (alternate) days of a month in winter (average temperature 10°C) and a month in summer (average temperature 30°C).

Consider the following statements based on the data shown above:
(i) Over the given months, the difference between the maximum and the minimum pollutant concentrations is the same in both winter and summer.
(ii) There are at least four days in the summer month such that the pollutant concentrations on those days are within 1 ppm of the pollutant concentrations on the corresponding days in the winter month.

Which one of the following options is correct?
(A) Only i
(B) Only ii
(C) Both i and ii
(D) Neither i nor ii

Key: (B)

Exp: The difference between the maximum and the minimum pollutant concentrations
(i) in winter = 8-0=8 ppm,
(ii) in summer = 10.5-1.5=9 ppm
∴ (i) is false & (ii) is correct from the graph.

9. All people in a certain island are either ‘Knights’ or ‘Knaves’ and each person knows every other person’s identity. Knights NEVER lie, and knaves ALWAYS lie.
P says “Both of us are knights”. Q says “None of us are knaves”.
Which one of the following can be logically inferred from the above?
(A) Both P and Q are knights
(B) P is a knight; Q is a knave
(C) Both P and Q are knaves
(D) The identities of P, Q cannot be determined

Key: (D)
10. X bullocks and Y tractors take 8 days to plough a field. If we halve the number of bullocks and double the number of tractors, it takes 5 days to plough the same field. How many days will it take X bullocks alone to plough the field?

(A) 30  (B) 35  (C) 40  (D) 45

Key:  (A)

Exp: Given Number of days required that X bullocks and Y tractors to plough a field = 8 days → (1) (i.e, $X + Y \rightarrow 8D \Rightarrow 8X + 8Y \rightarrow 1$ day) Number of days required that $\frac{X}{2}$ bullocks and 2Y tractors to plough field = 5 → (2) 

\[
\text{i.e, } \frac{X}{2} + 2Y \rightarrow 5D \Rightarrow \frac{5}{2}X + 10Y \rightarrow 1D
\]

From (1) & (2); we have

\[
8X + 8Y = \frac{5X}{2} + 10Y \Rightarrow Y = \frac{11X}{4}
\]

\[
\therefore \text{From (1)} X + Y \rightarrow 8D \Rightarrow \frac{15X}{4} \rightarrow 8D \Rightarrow X \rightarrow 30 \text{ days}
\]